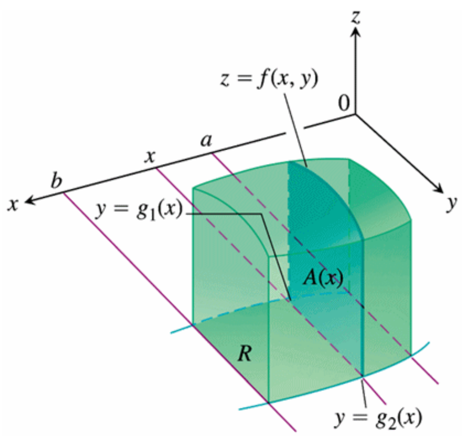
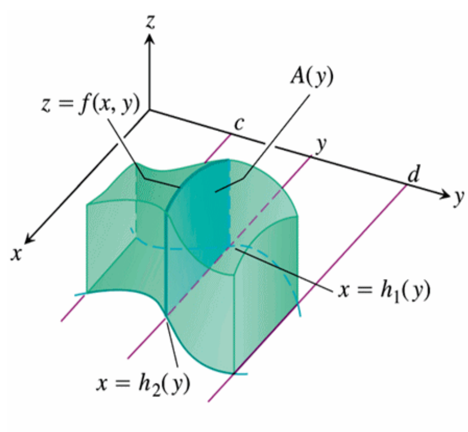
3D Integration

Double Riemann Sum If $f(x,y) \geq 0$ the double Riemann sum approximates the volume under the surface.

$$V \approx \sum_{i=1}^{m}\sum_{j=1}^{n} f(x_i^*, y_j^*)\underbrace{\Delta x \Delta y}_{\Delta A} \qquad \iint_R f(x,y)dA = \lim_{\substack{n\to\infty \\ m\to\infty}} \sum_{i=1}^{m}\sum_{j=1}^{n} f(x_i^*, y_j^*)\Delta A$$





To calculate the area of the vertical slice, $A(x)$:

$$A(x) = \int_{g_1(x)}^{g_2(x)} f(x,y)dy$$

Then sum the vertical slices as x goes from a to b:

$$\iint_R f(x,y)dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)dy\ dx$$

To calculate the area of the vertical slice, $A(y)$:

$$A(y) = \int_{h_1(y)}^{h_2(y)} f(x,y)dx$$

Then sum the vertical slices as y goes from c to d:

$$\iint_R f(x,y)dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)dx\ dy$$

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1. Evaluate $\displaystyle\int_1^2 \int_1^x 2x^2y^{-2} + 2y\, dy\ dx$

2. Sketch the region of integration in each of the problems below.

(a) $\displaystyle\int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} \int_{\cos(x)}^{\sin(x)} dy\ dx$

(b) $\displaystyle\int_{0}^{2} \int_{y^2}^{4} dx\ dy$

3. For each region sketched below create a double integral to calculate the area of the region R.