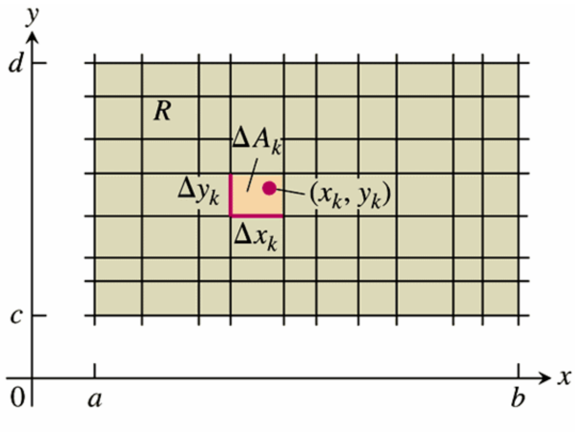
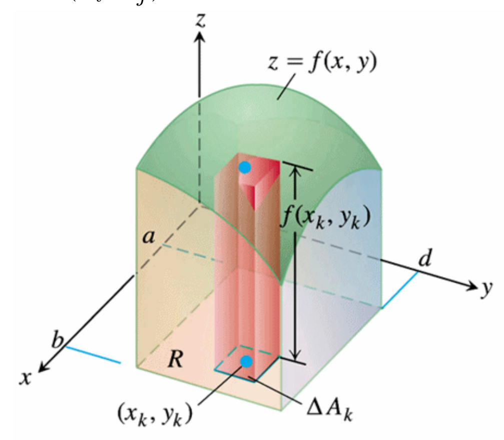
3D Integration

$f(x)$ on an interval $[a, b]$
$f(x_i^*)\Delta x$ little bit of area

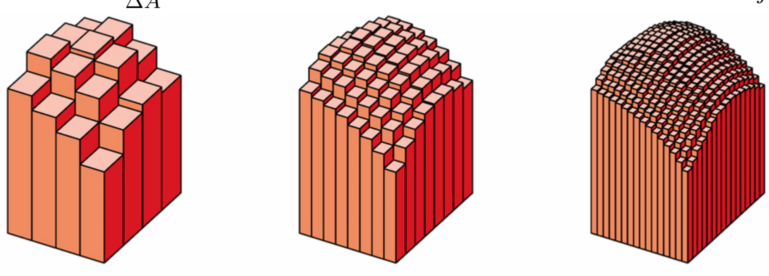
$f(x, y)$ on rectangle $R = [a, b] \times [c, d] =$
$f(x_i^*, y_j^*)\Delta x \Delta y$ little bit of volume





Double Riemann Sum If $f(x, y) \geq 0$ the double Riemann sum approximates the volume under the surface.

$$V \approx \sum_{i=1}^{m} \sum_{j=1}^{n} f(x_i^*, y_j^*) \underbrace{\Delta x \Delta y}_{\Delta A} \qquad \iint_R f(x,y)dA = \lim_{\substack{n\to\infty \\ m\to\infty}} \sum_{i=1}^{m} \sum_{j=1}^{n} f(x_i^*, y_j^*)\Delta A$$



(a) $n = 16$ (b) $n = 64$ (c) $n = 256$

While working in a group make sure you:

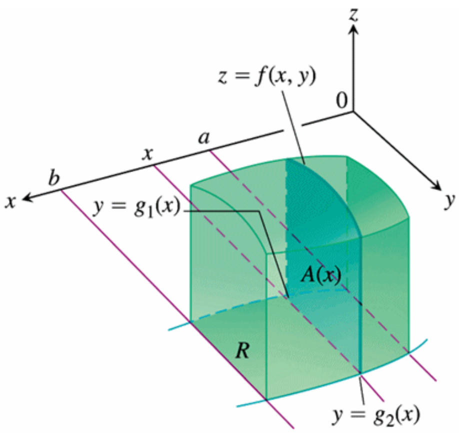
- Expect to make mistakes but be sure to reflect/learn from them!
- Are civil and are aware of your impact on others.
- Assume and engage with the strongest argument while assuming best intent.

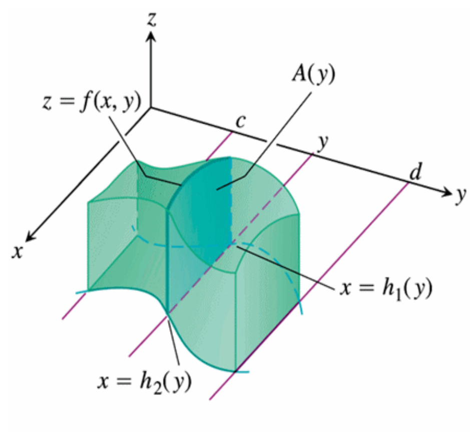
1. Let R be the rectangle $1 \leq x \leq 1.2$ and $2 \leq y \leq 2.4$. If the values for $f(x, y)$ are as specified below, find a Riemann sum approximation for $\iint_R f(x, y)dA$ with $\Delta x = 0.1$ and $\Delta y = 0.2$.

$y \setminus x$	1.0	1.1	1.2
2.0	5	7	10
2.2	4	6	8
2.4	3	5	6

2. Calculate $\int_0^4 \int_0^3 4x + 3y \, dy \, dx$

Fubini Theorem: If f is continous on the rectangle $R = \{(x, y) | a`x \leq b, c \leq y \leq d\}$, then $\iint_R f(x, y) dA = \int_a^b \int_c^d f(x, y) \, dy \, dx = \int_c^d \int_a^b f(x, y) \, dx \, dy$





To calculate the area of the vertical slice:

$$A(x) = \int_{g_1(x)}^{g_2(x)} f(x, y) dy$$

Then sum the slices as x goes from a to b:

$$\iint_R f(x, y) dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x, y) dy \; dx$$

To calculate the area of the vertical slice:

$$A(y) = \int_{h_1(y)}^{h_2(y)} f(x, y) dx$$

Then sum the slices as y goes from c to d:

$$\iint_R f(x, y) dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x, y) dx \; dy$$

3. For the region sketched below create a double integral to calculate the area of the region R.