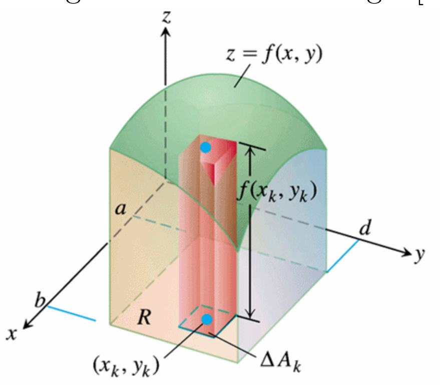
Integration over general regions

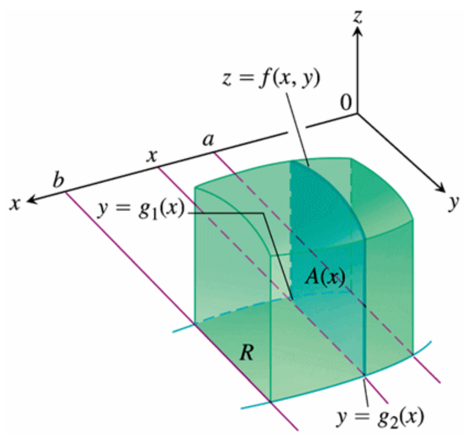
Double Riemann Sum If $f(x,y) \geq 0$ the double Riemann sum approximates the volume under the surface.

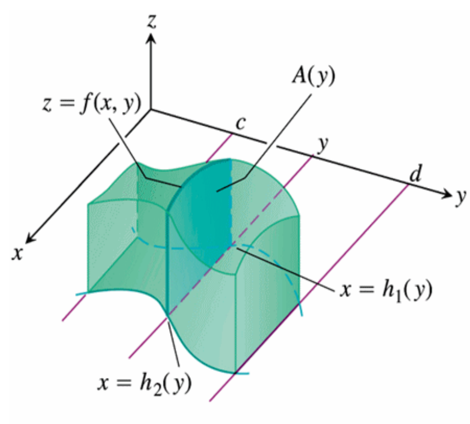
$$V \approx \sum_{i=1}^{m}\sum_{j=1}^{n} f(x_i^*, y_j^*)\underbrace{\Delta x \Delta y}_{\Delta A} \qquad \iint_R f(x,y)dA = \lim_{\substack{n\to\infty \\ m\to\infty}} \sum_{i=1}^{m}\sum_{j=1}^{n} f(x_i^*, y_j^*)\Delta A$$

If the region R was the rectangle $[a,b]\times[c,d]$, then we could use the iterated integral:



$$\iint_R f(x,y)dA = \int_a^b \int_c^d f(x,y)\,dy\,dx$$





To calculate the area of the vertical slice, $A(x)$:

$$A(x) = \int_{g_1(x)}^{g_2(x)} f(x,y)dy$$

Then sum the vertical slices as x goes from a to b:

$$\iint_R f(x,y)dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)dy\ dx$$

To calculate the area of the vertical slice, $A(y)$:

$$A(y) = \int_{h_1(y)}^{h_2(y)} f(x,y)dx$$

Then sum the vertical slices as y goes from c to d:

$$\iint_R f(x,y)dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)dx\ dy$$

Example: For each of the (familiar) regions sketched below, create a double integral to calculate the signed volume of the 3-dimensional solid region over the region R in the xy-plane and the surface $f(x, y) = yx$.