Improper Integrals

While working in a group make sure you:

- Expect to make mistakes but be sure to reflect/learn from them!
- Are civil and are aware of your impact on others.
- Assume and engage with the strongest argument while assuming best intent.

Definition 1. Define $\displaystyle\int_a^\infty f(x)\,dx = \lim_{b\to\infty}\int_a^b f(x)\,dx$ provided the integral & the limit exists.

Similarly for $\displaystyle\int_{-\infty}^b f(x)\,dx = \lim_{a\to-\infty}\int_a^b f(x)\,dx$.

And define $\displaystyle\int_{-\infty}^\infty f(x)\,dx = \lim_{a\to-\infty}\int_a^c f(x)\,dx + \lim_{b\to\infty}\int_c^b f(x)\,dx$.

We say the improper integral *converges* if the limit is finite. Otherwise, it *diverges*.

Definition 2. If f is continuous on $[a,b)$ and discontinuous at b then $\displaystyle\int_a^b f(x)\,dx = \lim_{s\to b^-}\int_a^s f(x)\,dx$ provided the limit exists.

1. Find:

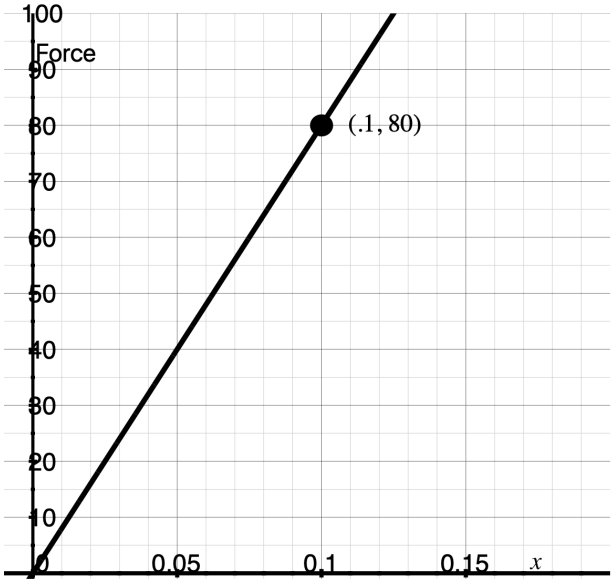
   $\displaystyle\int_{-\infty}^\infty \frac{1}{x^2+1}\,dx$ $\qquad\qquad$ $\displaystyle\int_0^3 \frac{1}{x-1}\,dx$

Work

Some physics notes:

- Force F, is an action or agency that causes a body of mass m to accelerate.
  - $F = m \cdot \frac{d^2s}{dt^2}$ where $s(t)$ is the position function
  - units are Newtons equal to kg $\cdot \frac{\text{m}}{\text{s}^2}$
  - units are sometimes pounds (lbs)
- Work W, is the amount of energy transferred by a force
  - $W = Fd$ if force F is constant
  - units are Joules equal to Newton meters
  - units are sometimes foot pounds

2. Consider the a non-constant force F (in Newtons) that is proportional to the distance moved x (in meters) graphed below. Find the work used to move from $x = 0$ to $x = .08$.



3. A force of 40N is required to hold a spring that has been stretched from its natural length of .1m to a length of .15 m. How much work is done in stretching the spring from .15m to .18m? Make use of Hooke's Law that says the force required to maintain a spring stretched x unites beyond its natural length is $F(x) = kx$ for some positive spring constant k.