Dot Products

Some Properties: If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are vectors in a vectors space V, and c is a scalar, then

- $\vec{a} \cdot \vec{a} = ||\vec{a}||^2$
- $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$
- $\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$
- $(c\vec{a}) \cdot \vec{b} = c(\vec{a} \cdot \vec{b}) = \vec{a} \cdot (c\vec{b})$
- $\vec{0} \cdot \vec{a} = \vec{0}$

While working in a group make sure you:

- Expect to make mistakes but be sure to reflect/learn from them!
- Are civil and are aware of your impact on others.
- Assume and engage with the strongest argument while assuming best intent.

1. Let $\vec{v} = 2\vec{i} + 2\vec{j} - \vec{k}$ and $\vec{w} = \langle 3, -2 \rangle$. Find (if possible):

   (a) $(\vec{v} \cdot \vec{j}) \cdot \vec{k}$

   (b) $||\vec{v}||$

   (c) $\vec{v} \cdot \vec{w}$

2. For each $\vec{v}$ and $\vec{w}$, determine if the two vectors are parallel, perpendicular, or neither. Note that "orthogonal" and "normal" are alternatives to the word "perpendicular".

   - $\vec{v} = \langle 2, 2, -1 \rangle$ and $\vec{w} = \langle 5, -4, 2 \rangle$

   - $\vec{v} = \langle 3, 7, -\frac{1}{2} \rangle$ and $\vec{w} = \langle -1, -\frac{7}{3}, \frac{1}{6} \rangle$

Cross Products

3. Find the following
   - $\overrightarrow{j} \times \overrightarrow{i}$
   - $\overrightarrow{3i} \times \overrightarrow{j}$

4. Cross product shows up in mechanics as torque τ. Specifically, $\tau = \overrightarrow{r} \times \overrightarrow{F}$ where $\overrightarrow{F}$ is the force applied at a radius of $\overrightarrow{r}$.

   Find the torque applied to a bolt if 50 lbs of vertical force is applied to the end of a 1 foot wrench attached to an axle with an angle of inclination of 60°.

Some nice properties of cross products: If $\overrightarrow{a}$, $\overrightarrow{b}$, and $\overrightarrow{c}$ are vectors and c is a scalar, then

- $\overrightarrow{a} \times \overrightarrow{b} = -\overrightarrow{b} \times \overrightarrow{a}$
- $(c\overrightarrow{a}) \times \overrightarrow{b} = c(\overrightarrow{a} \times \overrightarrow{b}) = \overrightarrow{a} \times (c\overrightarrow{b})$
- $\overrightarrow{a} \times (\overrightarrow{b} + \overrightarrow{c}) = \overrightarrow{a} \times \overrightarrow{b} + \overrightarrow{a} \times \overrightarrow{c}$
- $(\overrightarrow{a} + \overrightarrow{b}) \times \overrightarrow{c} = \overrightarrow{a} \times \overrightarrow{c} + \overrightarrow{b} \times \overrightarrow{c}$
- $\overrightarrow{a} \cdot (\overrightarrow{b} \times \overrightarrow{c}) = (\overrightarrow{a} \times \overrightarrow{b}) \cdot \overrightarrow{c}$
- $\overrightarrow{a} \times (\overrightarrow{b} \times \overrightarrow{c}) = (\overrightarrow{a} \cdot \overrightarrow{c})\overrightarrow{b} - (\overrightarrow{a} \cdot \overrightarrow{b})\overrightarrow{c}$