Cross Products

While working in a group make sure you:

- Expect to make mistakes but be sure to reflect/learn from them!
- Are civil and are aware of your impact on others.
- Assume and engage with the strongest argument while assuming best intent.

1. Find the following

    - $\vec{j} \times \vec{i}$

    - $3\vec{i} \times \vec{j}$

2. Find a unit vector orthogonal to both $\vec{i} + \vec{j} + \vec{k}$ and $2\vec{i} + \vec{k}$.

3. Find the angle between $\langle 1, -1, -1 \rangle$ and $\langle 1, -1, -1 \rangle \times \langle \frac{1}{2}, 1, \frac{1}{2} \rangle$.

Some nice properties of cross products: If $\vec{a}$, $\vec{b}$, and $\vec{c}$ are vectors and c is a scalar, then

- $\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$
- $(c\vec{a}) \times \vec{b} = c(\vec{a} \times \vec{b}) = \vec{a} \times (c\vec{b})$
- $\vec{a} \times (\vec{b} + \vec{c}) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$
- $(\vec{a} + \vec{b}) \times \vec{c} = \vec{a} \times \vec{c} + \vec{b} \times \vec{c}$
- $\vec{a} \cdot (\vec{b} \times \vec{c}) = (\vec{a} \times \vec{b}) \cdot \vec{c}$
- $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$

4. Find the area of the triangle PQR if $P(0, -2, 0)$, $Q(4, 1, -2)$, and $R(5, 3, 1)$.

5. Cross product shows up in mechanics as torque τ. Specifically, $\tau = \vec{r} \times \vec{F}$ where $\vec{F}$ is the force applied at a radius of $\vec{r}$.

Find the torque applied to a bolt if 50 lbs of vertical force is applied to the end of a 1 foot wrench attached to an axle with an angle of inclination of $60°$.