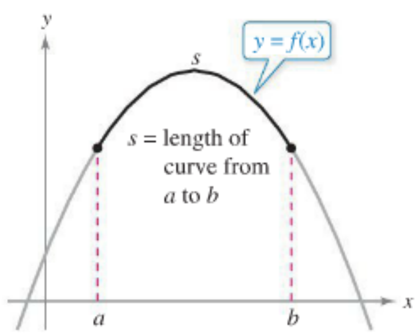
Calculus with Parametric Equations

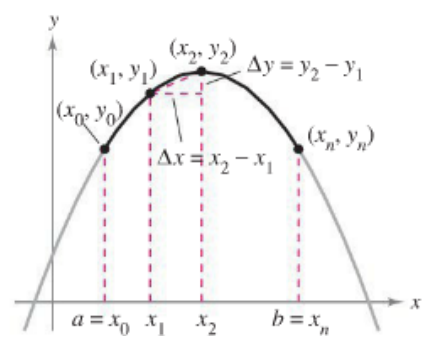
While working in a group make sure you:

- Expect to make mistakes but be sure to reflect/learn from them!
- Are civil and are aware of your impact on others.
- Assume and engage with the strongest argument while assuming best intent.

1. Consider the parametric equations $x(t) = r(t - \sin(t))$ and $y(t) = r(1 - \cos(t))$ where r is a fixed parameter. Find *where* the plane curve has a horizontal tangent line. Make sure that you have found them all!

2. Recall how we can use integral calculus to find the arc length of function by summing all the small line segments and then taking a limit as the $\delta x \to 0$.





In more detail: $s \approx \sum_{i=1}^{n} \sqrt{(\Delta x_i)^2 + (\Delta y_i)^2}$ where $\Delta x_i = x_i - x_{i-1}$.

Then through a bit of algebra (multiplying $(\Delta y_i)^2$ by $(\Delta x_i)^2/(\Delta x_i)^2$ mixed with some factoring), we find $s \approx \sum_{i=1}^{n} \sqrt{1 + (\Delta y_i/\Delta x_i)^2}\, \Delta x_i$.

Once we take the limit we have $s = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$

3. Set up the definite integral that gives the arc length resulting from tracing a point on a circle of radius 1 around a circle of radius 4. A picture is shown below and the corresponding parametric equations are: $x(t) = 5\cos(t) - \cos(5t)$ and $y(t) = 5\sin(t) - \sin(5t)$.