Optimizing in 3D

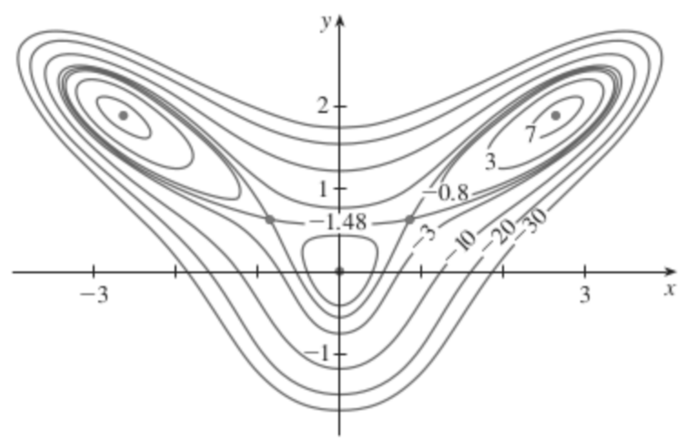
While working in a group make sure you:

- Expect to make mistakes but be sure to reflect/learn from them!
- Are civil and are aware of your impact on others.
- Assume and engage with the strongest argument while assuming best intent.

Some familiar definitions:

- A function of two variables has a *local maximum* at (a, b) if $f(x, y) \leq f(a, b)$ when (x, y) is near (a, b). The number $f(a, b)$ is called a *local maximum value*.
- A function of two variables has a *local minimum* at (a, b) if $f(a, b) \leq f(x, y)$ when (x, y) is near (a, b). The number $f(a, b)$ is called a *local minimum value*.
- Let (a, b) be a point in the domain D of a function f or two variables. Then $f(a, b)$ is the *absolute maximum value* of f on D if $f(x, y) \leq f(a, b)$ for all (x, y) in D.
- Let (a, b) be a point in the domain D of a function f or two variables. Then $f(a, b)$ is the *absolute minimum value* of f on D if $f(a, b) \leq f(x, y)$ for all (x, y) in D.

1. Consider the graph $f(x, y)$ whose contour lines are shown below.



   (a) Estimate the critical points on the graph.

   (b) Classify each critical point as a local maximum, local minimum, or neither.

   (c) Estimate the absolute maximum value of f.

Second Derivative Test: Suppose the second partial derivative of f are continuous on a disk with center (a, b), and that (a, b) is a critical point. Let

$$D = D(a, b) = f_{xx}(a, b)f_{yy}(a, b) - [f_{xy}(a, b)]^2$$

- If $0 < D$ and $0 < f_{xx}(a, b)$, then $f(a, b)$ is a local minimum value.
- If $0 < D$ and $f_{xx}(a, b) < 0$, then $f(a, b)$ is a local maximum value.
- If $D < 0$ then (a, b) is a saddle point of f.

2. Find the local maximum values of $f(x, y) = -x^3 + 4xy - 2y^2 + 1$

3. Find x, y, and z so that the sum is 42 and the sum of squares is a minimum.