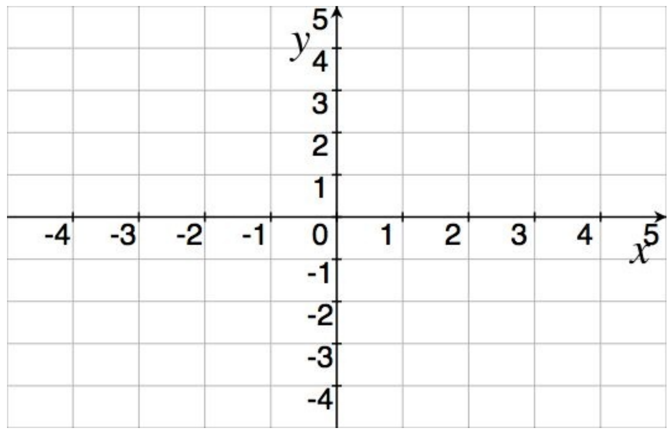
Parametric Equations

1. Consider the parametric equations $x(t) = t^2 - 4$ and $y(t) = \frac{t}{2}$.



   (a) Find the point on the plane curve when $t = 0$.

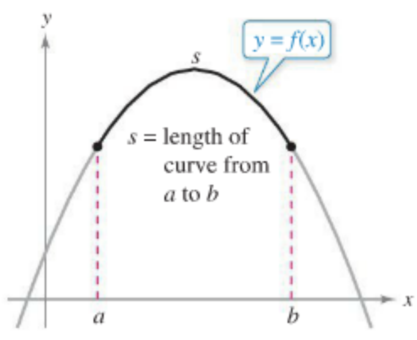
   (b) Sketch the graph of the plane curve as t varies from -2 to 3.

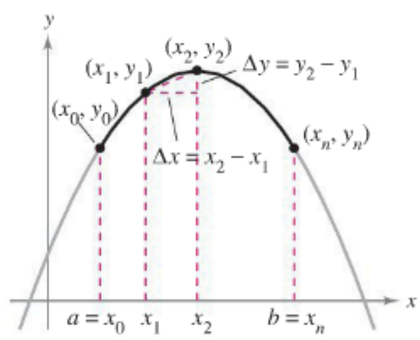
2. Eliminate the parameter in the parametric equations $x(t) = t^2 - 4$ and $y(t) = \frac{t}{2}$ to write the 'rectangular form' of the equation your graphed above.

Calculus with Parametric Equations

3. Consider the parametric equations $x(t) = (t - \sin(t))$ and $y(t) = (1 - \cos(t))$. Find *where* the plane curve has a horizontal tangent line. Make sure that you have found them all!

4. Recall how we can use integral calculus to find the arc length of function by summing all the small line segments and then taking a limit as the $\delta x \to 0$.





In more detail: $s \approx \displaystyle\sum_{i=1}^{n} \sqrt{(\Delta x_i)^2 + (\Delta y_i)^2}$ where $\Delta x_i = x_i - x_{i-1}$.

Then through a bit of algebra (multiplying $(\Delta y_i)^2$ by $(\Delta x_i)^2/(\Delta x_i)^2$ mixed with some factoring), we find $s \approx \displaystyle\sum_{i=1}^{n} \sqrt{1 + (\Delta y_i/\Delta x_i)^2}\,\Delta x_i$.

Once we take the limit we have $s = \displaystyle\int_a^b \sqrt{1 + (f'(x))^2}\,dx$