

# Quiz 8

Key

This is a two-stage quiz. During the first stage, use a one-sided 8.5" by 11" sheet of notes & calculator. You have 15 min. In the second stage, you are now welcome to use your books, notes, and students in the class to retake the same quiz. You have the remainder of the quiz time to write one solution (with everyone's name on it!!!) to be turned in for the group.

1. Consider the contour lines of the function  $f(x, y)$  shown below.

- (a) [4] Use Riemann sums to approximate the (signed) volume captured between  $f(x, y)$  & the  $xy$  plane over  $R$  where  $R$  is defined by  $2 \leq x \leq 5$ , and  $0 \leq y \leq 2$ . Clearly indicate how many rectangular prisms you are using & values used.

Integration tech #1

(+1) I'll use 6 prisms (outlined on contour graph)

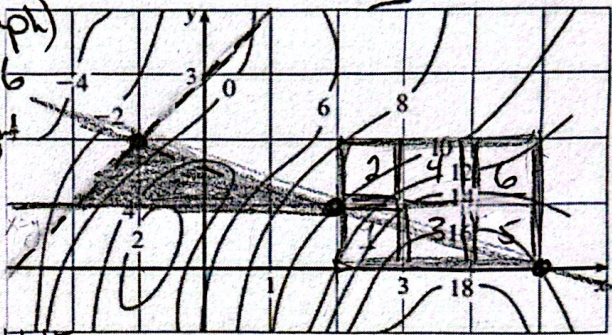
$$Vol_1 + Vol_2 + Vol_3 + Vol_4 + Vol_5 + Vol_6$$

(+2) recall  $Vol = \underbrace{\text{length}} \cdot \underbrace{\text{width}} \cdot \underbrace{\text{height}}$

(+1.5) Choose bottom left of box for height of prism

(+1.5)  $1 \cdot 1 \cdot 2 + 1 \cdot 1 \cdot 0 + 1 \cdot 1 \cdot 7 + 1 \cdot 1 \cdot 4 + 1 \cdot 1 \cdot 7 + 1 \cdot 1 \cdot 5 = 85$

There are lots of correct approx



- (b) [2] Consider  $\int_1^2 \int_{y-3}^{-3y+5} f(x, y) dx dy$ . Sketch and shade the region of integration on the graph with the contour lines above.

S15.2 #28

(+1.5)  $x = -3y + 5$   
 $\frac{1}{3}(x-5) = -y$  thru  $(-1, 2)$

(+1.5)  $x = y - 3$   
 $x + 3 = y$  thru  $(-1, 2)$

(+1.5)  $y = 2$  and  $y = 1$

Webb 11.1 #52

- [2] Find  $\int_0^4 \int_0^4 x \sec^2(y) dx dy$

Lots of ways to do this problem

technology

OR algebra

treat  $y$  as const

int/ply n limits

(+1.5) undefined when

(+1.5) b/c @  $y = \pi/2$   
 $\sec^2(y)$  is not defined

$$\int_0^4 \left[ \frac{1}{2} x^2 \sec^2(y) \right]_0^4 dy = \int_0^4 \frac{1}{2} (4)^2 \sec^2(y) dy = \int_0^4 8 \sec^2(y) dy$$

Note  $\sec(y)$  is not defined at  $\pi/2$   
 $\Rightarrow$  cannot use FTC - ugh! (+1.5)

3. [2] Find a formula for the general term  $a_n$  where  $\{a_n\}_{n=-1}^{\infty} = \{-\frac{1}{5}, \frac{1}{10}, -\frac{1}{15}, \frac{1}{20}, \dots\}$

$$\frac{(-1)^1}{5 \cdot 1}, \frac{(-1)^2}{5 \cdot 2}, \frac{(-1)^3}{5 \cdot 3}$$

$$a_n = \frac{(-1)^n}{5(n+2)} = \frac{(-1)^n}{5n+10}$$

(+1.5) negative one

inc by 5 each time (+1.5)