Exam 2 # TMath 126 Spring 2025

As a reminder, you are welcome to use a non-internet accessing calculator (which includes Desmos Test Mode) and a one-sided 8.5" by 11" sheet of notes. Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

1. Let $\beta(x,y) = \dfrac{x-2}{x^2y - 4y}$. Find the following, if possible.

   (a) [1] $\beta(11, 2)$

   (b) [2] (§14.2 #46) a point *not* in the domain of β.

   (c) [2] (LimitActivity#3) $\lim_{(x,y)\to(2,3)} \beta(x,y)$

   (d) [2] (Quiz5#2) $\beta_y(x,y)$

   (e) [1] (WebHW14.4#1) $\beta_y(1,4)$

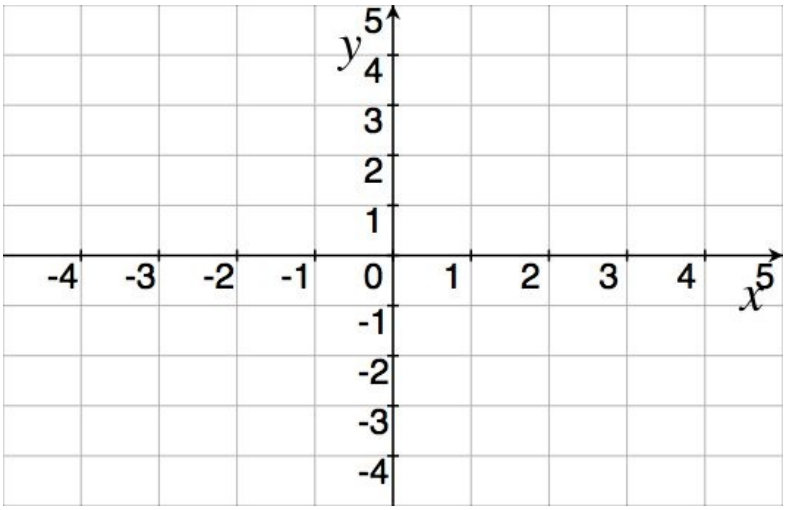
2. [5] Draw the contours of a function f of two variables (x, y), that satisfies *all* of the following:

   (a) $f(2,1) = 5$

   (b) $f_x(2,1) \approx 2$

   (c) $f_y(2,1) \approx -1$

   (d) $(-2,-3)$ is a saddle point



3. Let $\vec{u} = \frac{3}{\sqrt{10}}\vec{i} - \frac{1}{\sqrt{10}}\vec{j}$ and let α be a differentiable function with

   - $\alpha(-3,2) = 4$
   - $\alpha_x(-3,2) = -1$
   - $\alpha_y(-3,2) = 3$

   (a) [3] (TangentActivity #1) Find a linearization for α when $x = -3$ and $y = 2$.

   (b) [2] (WebHW§14.4 #3) Use your linearization to approximate $\alpha(-3.2, 1.9)$.

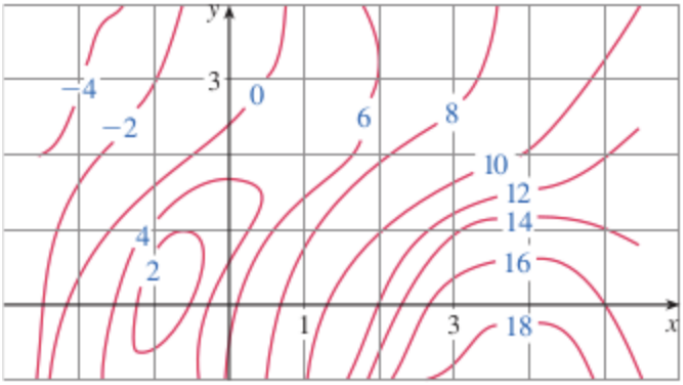
   (c) [2] (PracticeExam#4) Find $D_{\vec{u}}\alpha(-3,2)$.

4. (§10.1 #58) Let g have the contour lines shown on the right.

   (a) [1] (OptimizeActivity#1) Identify a critical point.

   (b) [2] (§14.6 #26) Sketch the direction of the gradient vector at $(2, 3)$

   (c) [2] (PracticeExam#2) Identify a point (a, b) where $g_x(a, b) < 0$.



5. A particular hill's elevation is well modeled by $z = 2000 - 0.005x^2 - .01y^2$ where x, y, and z are measured in meters. The positive x-axis points east and the positive y-axis points north.

   (a) [1] What is your elevation at $x = 160$ and $y = 160$?

   (b) [2] (§14.3#2) What is the meaning of $\frac{\partial z}{\partial y}$ in terms of walking and elevation?

   (c) [3] If you walk east, from the point $x = 160$ and $y = 160$, do you ascend or descend? Provide justification.

   (d) [3] (WebHW14.3 #6) When starting from $x = 160$ and $y = 160$, in which direction should you go to increase your elevation the quickest with one step?

6. [6] (Suggested§14.8 #55) A grain silo is to be built by attaching a hemispherical roof and a flat floor onto a circular cylinder. The material (surface area) is restricted to 800 square meters. We want to find the dimensions of the silo that maximize volume. Outline the solution and if you use chapter 14 techniques, make sure to:

   (a) define variables used

   (b) circle the function that needs to be optimized

   (c) box any systems of equations that need to be solved (but you do not need to solve them!), and

   (d) explain how you would verify you have a maximum.