Note for future: be sure you know how to use your tex before exam.

Key

$/10$

Exam 1 — **TMath 126** — Spring 2025

As a reminder, you are welcome to use a non-internet accessing calculator (which includes Desmos Test Mode) but no books, other notes, or peers. Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

1. [6] TRUE/FALSE: Write True in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, write False and provide a counterexample or brief justification.

   (a) (PracticeExam#1) If $\vec{v}$ and $\vec{w}$ are vectors in $\mathbb{R}^3$ so that $\vec{v} \times \vec{w} = 0$ (that is, the cross product of vectors v and w), then $\vec{v}$ or $\vec{w}$ equal $\langle 0, 0, 0\rangle$.

   def of cross (+1)
   sense/notation (+.5)
   counterex/logic (+1)

   False (+.5)

   $\vec{v}\times\vec{w}$ should be a vector, 0 is a scalar, so $\vec{v}\times\vec{w}=0$ does not make sense.

   Also consider $\langle 1,0,0\rangle \times \langle 1,0,0\rangle = \langle 0,0,0\rangle$

   The cross product equalling $\vec{0}$ => vectors are parallel.

   (b) (§10.2#14) If $\vec{r}(t) = \langle 3^t, t\cos(2t)\rangle$, then the line tangent to $\vec{r}(1)$ is:

$$y - 1 = \frac{-2t\sin(2t) + \cos(2t)}{3^t(\ln 3)}(x-1)$$

   note this would be $(3^1, 1\cos(2\cdot 1))$ or $(3, \cos(2))$ which are not the points entered?

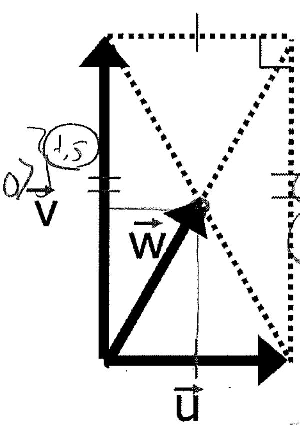
   line (+1)
   sense/notation (+.5)
   counter ex/logic (+1)

   False (+.5)

   Lines look like $y - y_0 = m(x - x_0)$ where m is a slope/number.

   Here we have a complicated function of x, y and t

2. Let $\vec{u}$, $\vec{v}$, and $\vec{w}$ be the vectors shown on the right. Assume $\vec{u}$ is a unit vector and that $||\vec{v}|| = \sqrt{3}$.



   (a) [2] (WebHW12.3#5) Find $\vec{u} \cdot \vec{w}$

   Set: $\vec{i} \rightarrow$, $\uparrow \vec{j}$ (+.5)

   So $\vec{u} = \langle 1,0,0\rangle$, $\vec{w} = \langle \frac{1}{2}, \frac{\sqrt{3}}{2}, 0\rangle$ (+.5)

   $\Rightarrow \vec{u}\cdot\vec{w} = \langle 1,0,0\rangle \cdot \langle \frac{1}{2}, \frac{\sqrt{3}}{2}, 0\rangle$

   $= \frac{1}{2}$

   (+.5) dot product
   (+.5) computation

   (b) [2] (Quiz2#2) Find $\vec{u} \times \vec{v}$

   vector answer (+.5)
   scalar (+.5)
   def of cross (+.5)
   notation (+.5)

   Same axis as above

   $\vec{u} = \langle 1,0,0\rangle$, $\vec{v} = \langle 0, \sqrt{3}, 0\rangle$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 0 & 0 \\ 0 & \sqrt{3} & 0 \end{vmatrix} = 0\vec{i} - 0\vec{j} + \sqrt{3}\vec{k}$$

   So $\sqrt{3}\vec{k}$

   OR

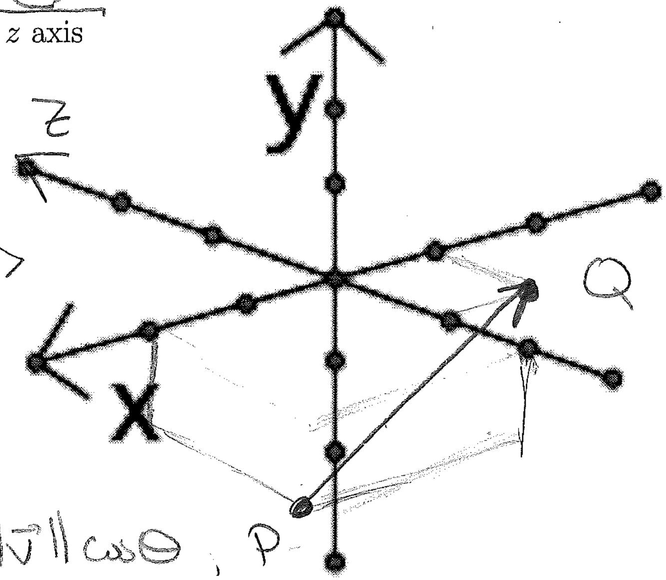
   $\|\vec{u}\times\vec{v}\|$ area of rect. $\|u\| \cdot \|v\| = \sqrt{3}\cdot 1$

   perpendicular using right hand rule so

   $\sqrt{3}$ out of the page

3. Consider the points $P(2,-1,-2)$ and $Q(-1,0,-1)$. Let $\vec{v} = \langle 1,3,0\rangle$.

(+.5)



(a) [2] (VectorActivity#1) Label the positive z axis and then plot the vector $\overrightarrow{PQ}$.

direction (+.5) (+1)

(b) [2] (Quiz1#1)
Find the components of $\overrightarrow{PQ}$.

$\langle -1-2, 0-(-1), -1-(-2)\rangle = \langle -3, 1, 1\rangle$

notation (+.5) dif (+.5) order (+.5) got it (+.5)

(c) [2] (WebHW12.3#6)
Find the angle $\overrightarrow{PQ}$ makes with $\vec{v}$.

(+.5) Recall $\overrightarrow{PQ} \cdot \vec{v} = \|\overrightarrow{PQ}\| \|\vec{v}\| \cos\theta$, P

need to find θ

(+1) $\langle -3,1,1\rangle \cdot \langle 1,3,0\rangle = \sqrt{9+1+1}\sqrt{1+9+0}\cos\theta$

$\cancel{-3}+\cancel{3}+0 = \sqrt{11}\sqrt{10}\cos\theta$

(+.5)

$\Rightarrow 0 = \sqrt{11}\sqrt{10}\cos\theta \Rightarrow \cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2}$

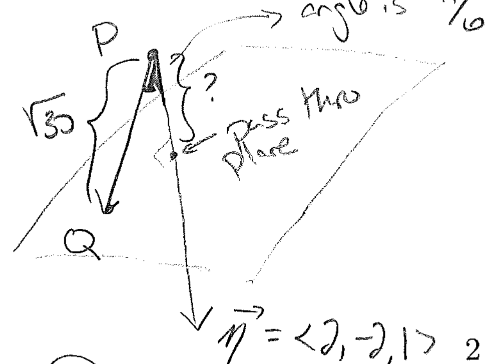
4. Use the following information below for the following questions.

- The object A is given by $0 = \langle 2,-2,1\rangle \cdot (\langle x,y,z\rangle - \langle 0,1,0\rangle)$.
- Let the point $Q \in \mathbb{R}^3$ satisfy the equation of A.
- Let P be another point such that $\|\overrightarrow{PQ}\| = \sqrt{30}$,
- the angle between $\langle 2,-2,1\rangle$ and $\overrightarrow{PQ}$ is $\frac{\pi}{6}$ radians.

(a) [1] (LinesPlanesActivity) Identify if A is a line, a plane, or some other 3D object.

Plane ? normal vector is $\langle 2,-2,1\rangle$
(in 3D)

(b) [3] (WrittenHW12.5 #72) Find the distance between the point P and A.



want to find ?
Sohcahtoa
$\cos \frac{\pi}{6} = \frac{?}{\sqrt{30}}$ (+1)

$\Downarrow$

$\sqrt{30}\cos\frac{\pi}{6} = ?$

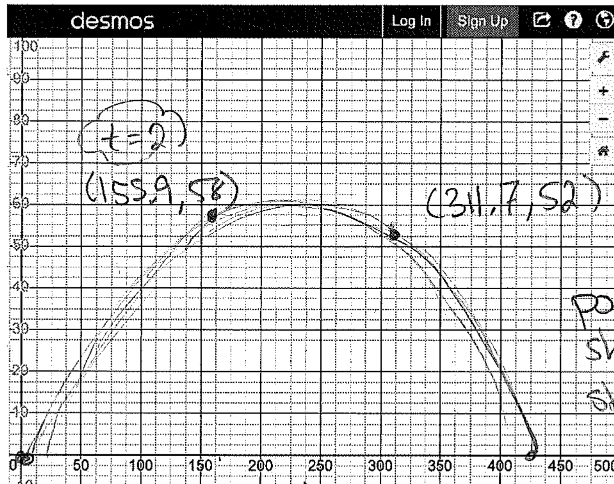
picture (+1)

$\frac{\sqrt{30}\sqrt{3}}{2} = ? = \frac{3\sqrt{10}}{2} \approx 4.74$

alg (+1)

5. (§10.1 #58) The path of a projectile is modeled by $x = 90t\cos(30°)$ and $y = 90t\sin(30°) - 8t^2$, in degrees where x and y are measured in feet.

(a) [2] Sketch the path of the projectile



points (+.5)
shape (+1)
start (+.5)

(b) [3] Find how far the projectile travels horizontally before hitting the ground.

start (+.5)

(+1) [ie when $y = 0$ what is x

(+1) $90t\sin(30°) - 8t^2 = 0$

$t(90\sin(30°) - 8t) = 0$

$t = 0$ or $90\sin(30°) - 8t = 0$

$\Rightarrow t = \dfrac{90\sin(30°)}{8}$

$= \dfrac{45}{8} \approx 5.625$ So x is $90\left(\frac{45}{8}\right)\cos(30)° \approx 438$] (+.5)

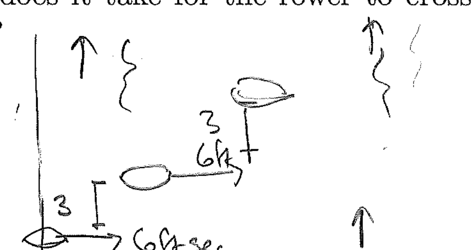
6. (WordProblem #2) A person wants to row her kayak across a river that is 1800 ft wide. She can row (when in the water) 6 ft/s.

(a) [1] If there is no current, how long does it take for the rower to cross the river?

(+.5) rate · time = dist $\Rightarrow$ 6 ft/s · t = 1800 $\Rightarrow$ 300 sec (+.5) = 5 min

(b) [1] If there is a current of 3 ft/sec, and the kayak is aimed straight ahead, how long does it take for the rower to cross the river and how far down stream are they?

picture (+.5)



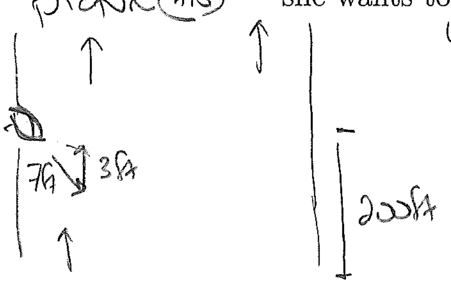
So travel 3ft down river each second.

Takes 300 sec to cross

$\Rightarrow$ 300 sec · 3 ft/sec = 900 ft (+.5)

(c) [3] If the person needs to go 200 feet *upstream*, what angle does she need to keep the front of the kayak to make a straight line to her destination and land where she wants to?

picture (+.5)



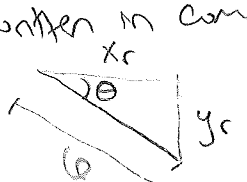
Want to find θ so

$\overrightarrow{\text{row rate}} \cdot t + \overrightarrow{\text{river rate}} \cdot t = \langle 1800, -200 \rangle$ (+1)

$\langle 6\cos\theta, 6\sin\theta \rangle \cdot t + \langle 0, 3 \rangle t = \langle 1800, -200 \rangle$

$\Rightarrow \begin{cases} 6t\cos\theta + 0t = 1800 \\ 6t\sin\theta + 3t = -200 \end{cases}$ system of equations

written in components



Sohcahtoa

$\cos\theta = \dfrac{x_r}{6}$ $\sin\theta = \dfrac{y_r}{6}$

(+1)

algebraically

$t = \dfrac{1800}{6\cos\theta}$ from (1)

sub into equation (2)

$6\left(\dfrac{300}{\cos\theta}\right)\sin\theta + 3\left(\dfrac{300}{\cos\theta}\right) = -200$

use trig identities

technology

intersect when

$t \approx 371$ sec

$\theta \approx -36.1°$ (+.5)

7. (Quiz2#8) Let B be defined by $\langle 3+4t, -2t, -1+t\rangle$ where $t \in \mathbb{R}$.

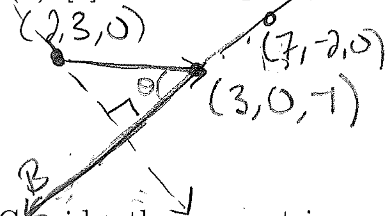
(a) [1] (LinesPlanesActivity) Identify if B is a line, a plane, or some other 3D object.

$\langle 3,0,-1\rangle + t\langle 4,-2,1\rangle = \langle x,y,z\rangle$ line

point direction

would have been easier if a plane. Up 3 pts. just giving everyone full points b/c of my error

(b) [2] Find an equation of a line that is perpendicular to B and goes through $(2,3,0)$.



$\langle 1,-3,-1\rangle \cdot \langle 4,-2,1\rangle = \sqrt{11}\sqrt{21}\cos\theta$

$9 = \sqrt{11}\sqrt{21}\cos\theta \Rightarrow \theta \approx 54^\circ$

8. Consider the parametric curve defined by $x(t) = t^2+1$, $y(t) = 4\sqrt{t}$, and $z(t) = e^{t^2-t}$.

(a) [2] (Web13.1 #4) Which of the two sketches provided are the graph of the given parametric equation? Justify your answer.

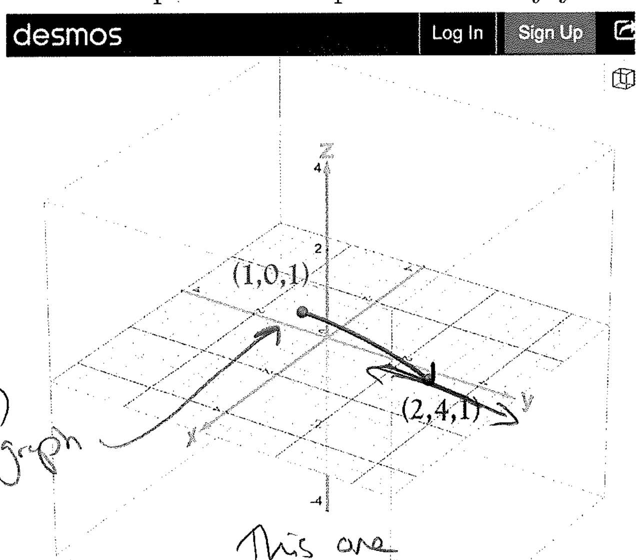
$t=0$

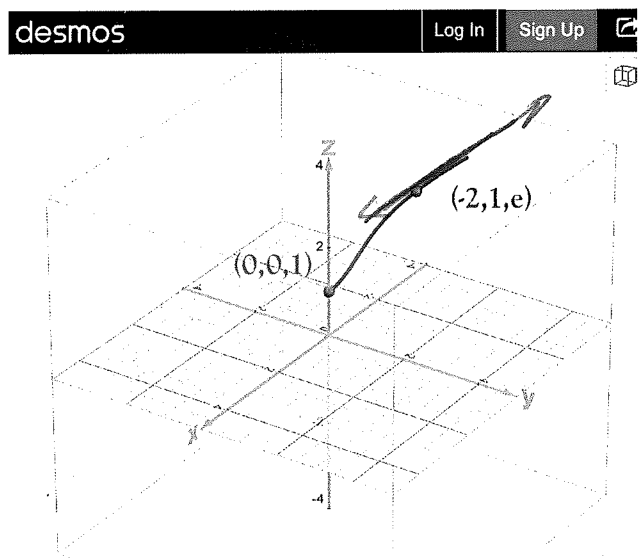
$x(0)=1$

$y(0)=0$

$z(0)=1$

so $(1,0,1)$ is on the graph





This one

(b) [1] Sketch a *line* tangent to one of the above curves at the point farther from the z axis. either works as long as a line.

(c) [4] (Suggested §13.2 #25) Find an equation for a line that is tangent to the parametric curve defined algebraically above and passes through the green point.

Looking for a line: $\langle x,y,z\rangle = \langle x_0,y_0,z_0\rangle + t\vec{d}$ (+.5)

Thru $(2,4,1)$ (+.5) let $(2,4,1)$ be P

$\vec{d}$ = instantaneous rate of change @ P

(+.5) $= \left\langle \frac{dx}{dt}\Big|_P, \frac{dy}{dt}\Big|_P, \frac{dz}{dt}\Big|_P \right\rangle$

$= \langle 2(1), 4(\tfrac{1}{2})(1)^{-1/2}, e^0(2-1)\rangle$

(+1.5) $dx/dt = 2t$

$dy/dt = 4(\tfrac{1}{2})t^{-1/2}$

$dz/dt = e^{t^2-t}(2t-1)$

(+.5) what t value when @ P?

$2 = t^2+1 \Rightarrow 1 = t^2 \Rightarrow t = \pm 1$

$4 = 4\sqrt{t} \Rightarrow t = 1$

note $e^{1^2-1} = e^0 = 1$ ✓

plug in (+.5)

So $\langle x,y,z\rangle = \langle 2,4,1\rangle + t\langle 2,2,1\rangle$