Exam 1 # TMath 126 Spring 2025

As a reminder, you are welcome to use a non-internet accessing calculator (which includes Desmos Test Mode) but no books, other notes, or peers. Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

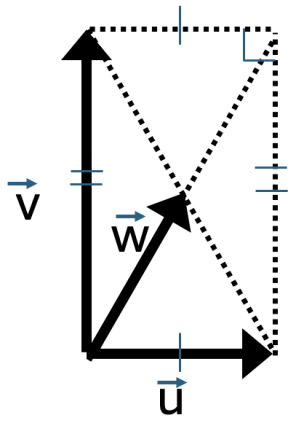
1. [6] TRUE/FALSE: Write True in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, write False and provide a counterexample or brief justification.

   (a) (PracticeExam#1) If $\vec{v}$ and $\vec{w}$ are vectors in $\mathbb{R}^3$ so that $\vec{v} \times \vec{w} = 0$ (that is, the cross product of vectors v and w), then $\vec{v}$ or $\vec{w}$ equal $\langle 0, 0, 0 \rangle$.

   (b) (§10.2#14) If $\vec{r}(t) = \langle 3^t, t\cos(2t) \rangle$, then the line tangent to $\vec{r}(1)$ is:

   $$y - 1 = \frac{-2t\sin(2t) + \cos(2t)}{3^t(\ln 3)}(x - 1)$$

2. Let $\vec{u}$, $\vec{v}$, and $\vec{w}$ be the vectors shown on the right. Assume $\vec{u}$ is a unit vector and that $||\vec{v}|| = \sqrt{3}$.

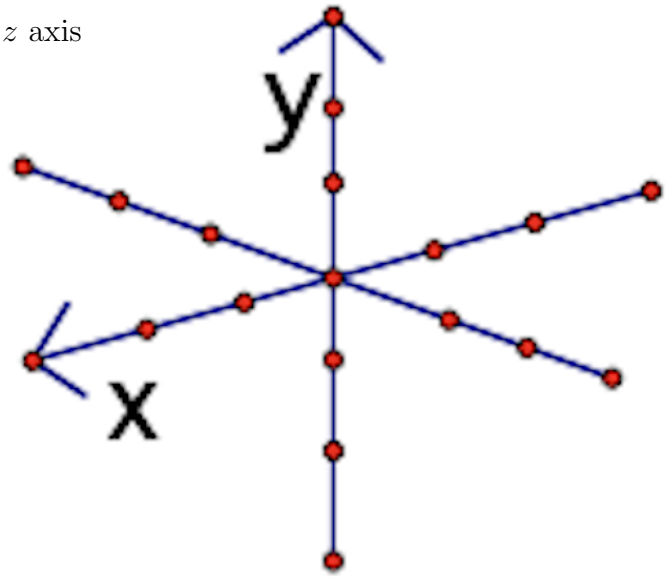


   (a) [2] (WebHW12.3#5) Find $\vec{u} \cdot \vec{w}$

   (b) [2] (Quiz2#2) Find $\vec{u} \times \vec{v}$

3. Consider the points $P(2,-1,-2)$ and $Q(-1,0,-1)$. Let $\overrightarrow{v} = \langle 1,3,0 \rangle$.

   (a) [2] (VectorActivity#1) Label the positive z axis and then plot the vector $\overrightarrow{PQ}$



   (b) [2] (Quiz1#1) Find the components of $\overrightarrow{PQ}$.

   (c) [2] (WebHW12.3#6) Find the angle $\overrightarrow{PQ}$ makes with $\overrightarrow{v}$.

4. Use the following information below for the following questions.

   - The object A is given by $0 = \langle 2,-2,1 \rangle \cdot (\langle x,y,z \rangle - \langle 0,1,0 \rangle)$.
   - Let the point $Q \in \mathbb{R}^3$ satisfy the equation of A.
   - Let P be another point such that $||\overrightarrow{PQ}|| = \sqrt{30}$,
   - the angle between $\langle 2,-2,1 \rangle$ and $\overrightarrow{PQ}$ is $\frac{\pi}{6}$ radians.

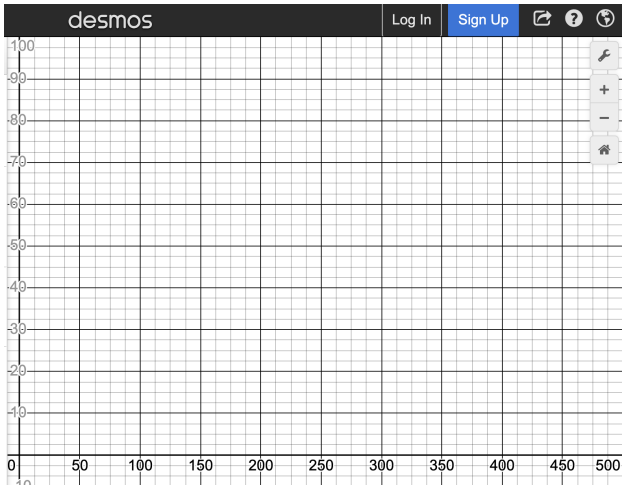
   (a) [1] (LinesPlanesActivity) Identify if A is a line, a plane, or some other 3D object.

   (b) [3] (WrittenHW12.5 #72) Find the distance between the point P and A.

5. (§10.1 #58) The path of a projectile is modeled by $x = 90t\cos(30°)$ and $y = 90t\sin(30°) - 8t^2$, in degrees where x and y are measured in feet.

   (a) [2] Sketch the path of the projectile

   (b) [3] Find how far the projectile travels horizontally before hitting the ground.



6. (WordProblem #2) A person wants to row her kayak across a river that is 1800 ft wide. She can row (when in the water) 6 ft/s.

   (a) [1] If there is no current, how long does it take for the rower to cross the river?

   (b) [1] If there is a current of 3 ft/sec, and the kayak is aimed straight ahead, how long does it take for the rower to cross the river and how far down stream are they?

   (c) [3] If the person needs to go 200 feet *upstream*, what angle does she need to keep the front of the kayak to make a straight line to her destination and land where she wants to?

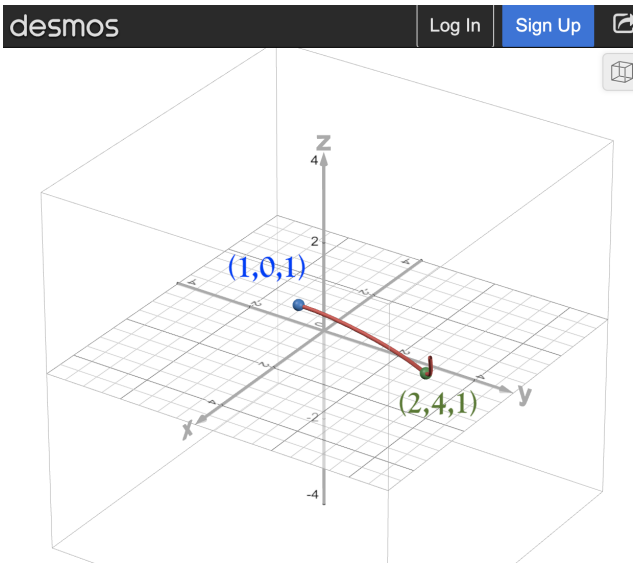
7. (Quiz2#8) Let B be defined by $\langle 3+4t, -2t, -1+t\rangle$ where $t \in \mathbb{R}$.

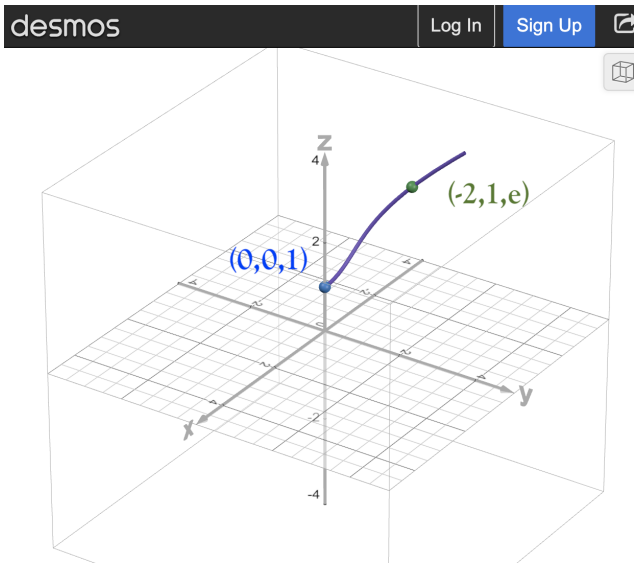
   (a) [1] (LinesPlanesActivity) Identify if B is a line, a plane, or some other 3D object.

   (b) [2] Find an equation of a line that is perpendicular to B and goes through $(2, 3, 0)$.

8. Consider the parametric curve defined by $x(t) = t^2 + 1$, $y(t) = 4\sqrt{t}$, and $z(t) = e^{t^2-t}$.

   (a) [2] (Web13.1 #4) Which of the two sketches provided are the graph of the given parametric equation? Justify your answer.





   (b) [1] Sketch a *line* tangent to one of the above curves at the point farther from the z axis.

   (c) [4] (Suggested §13.2 #25) Find an equation for a line that is tangent to the parametric curve defined algebraically above and passes through the green point.