Key

Exam 2 | TMath 126 | Autumn 2025

As a reminder, you are welcome to use a non-internet accessing calculator (which includes Desmos Test Mode) and one side of notes. Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

1. Choose *ONE* of the following functions to compute the questions below. That is, do parts (a), (b), and (c) with either the function α or β, not both.

$$z = \alpha(x, y) = y^2 \ln(x) \qquad \text{XOR} \qquad z = \beta(x, y) = \cos(\pi x y)$$

(a) [3] (3DderivativeActivity #3) Find $\frac{\partial z}{\partial x}$ treat y as const (+1) start (+.5)

$\frac{\partial z}{\partial x} = y^2 \cdot \underbrace{\frac{1}{x}}_{(+.5)}$ notation (+1) | $\partial z/\partial z = \underbrace{[-\sin(\pi x y)]}_{(+.5)} \cdot \pi y$ chain (+1)

(b) [1] (WebHW14.4 #2) Find $z_x(1, 4)$. plug in $x=1$ and $y=4$ to (a) (+1)

$\left.\frac{\partial z}{\partial x}\right|_{(1,4)} = (4)^2 \cdot \frac{1}{1} = 16$ | $[-\sin(\pi(1)(4))] \cdot \pi \cdot 4 = 0$

(c) [3] (WebHW14.6 #2) Let $\vec{u} = \langle 2, -4 \rangle$, find $D_{\vec{u}}(1, 4)$

(+1) { Note $\|\vec{u}\| = \sqrt{2^2 + (-4)^2} = \sqrt{4+16} = \sqrt{20}$ so use $\langle \frac{2}{\sqrt{20}}, \frac{-4}{\sqrt{20}} \rangle$

(+.5) { $D_{\vec{u}}(1,4) = \nabla z(1,4) \cdot \langle \frac{2}{\sqrt{20}}, \frac{-4}{\sqrt{20}} \rangle$

(+1) { $\partial z/\partial y = 2y \ln(x)$ $\left.\partial z/\partial y\right|_{(1,4)} = 8\ln(1)$

(+.5) { $\langle 16, 0 \rangle \cdot \langle \frac{2}{\sqrt{20}}, \frac{-4}{\sqrt{20}} \rangle = \frac{32}{\sqrt{20}}$

2. Let $f(x, y) = \sqrt{x - 2y}$

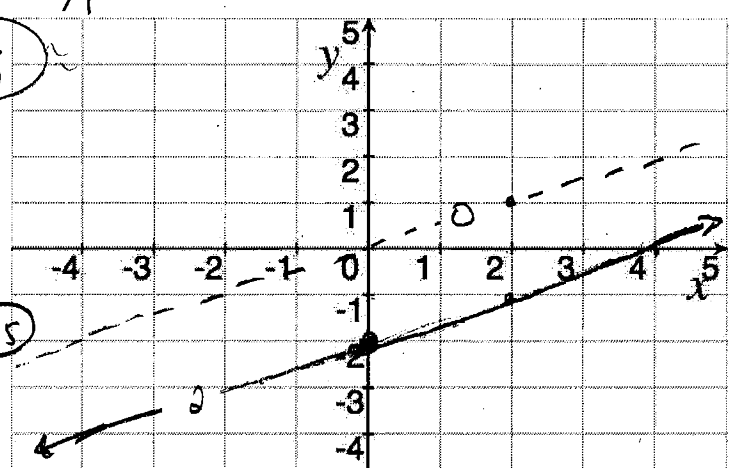
(a) [2] (WebHW14.1#2) Find the domain of f.

Notation (+.5)

inputs (+.5)

(+.5) stuff under square root ≥ 0

$\Rightarrow x - 2y \geq 0$ or $x \geq 2y$ (+.5)



(b) [3] (3DFunctionActivity #2) Sketch the contour lines/level curves for $f(x, y) = 2$ and $f(x, y) = 0$. Label them!

(+.5) set z to 0 and 2

(+1) alg

(+1) graph (complete)

(+.5) start

$2 = \sqrt{x - 2y}$

$4 = x - 2y$

$2y = x - 4$

$y = \frac{1}{2}x - 2$

$0 = \sqrt{x - 2y}$

$0 = x - 2y$

$2y = x$

$y = \frac{1}{2}x$

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3. [5] (Quiz5 #1) Create a function (+1) $g(x, y) = z$, with contour ~~lines/~~level curves that satisfy *all* of the following

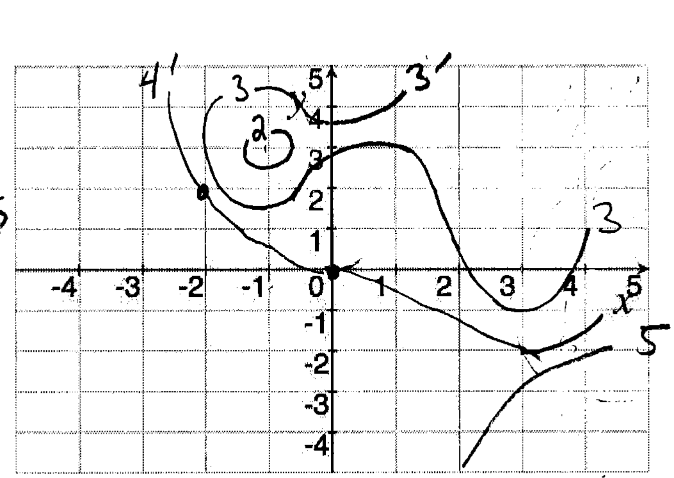
Note: there are MANY correct answers

(+1) (a) $f(-2, 2) = f(0, 0) = 2$

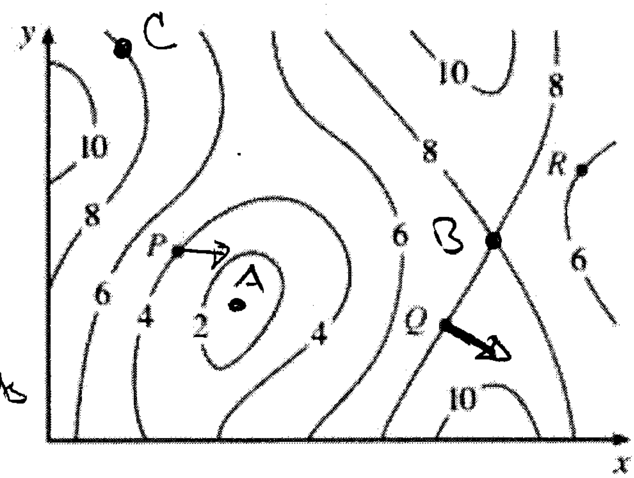
(+1) (b) $f_x(3, -2) > 0$ $\longrightarrow$

(+1) (c) $f_y(3, -2) \approx -1$ $\uparrow$

(+1) (d) $(-1, 3)$ is a local minimum



4. The contour map of a function $h(x, y)$ is given below.



(a) [2] (OptimizeAct #1)

(+1) $\rightarrow$ Identify a critical point.

(+1) $\rightarrow$ Classify the point as a local min, local max, or saddle point.

A is local min

B is a saddle

(b) [1] (WebHW14.3 #1) Is the partial of h relative to x at P positive or negative?

(+1) negative as x inc, outputs go down

(c) [1] (§14.6 #26) Sketch the direction of ∇h at the point Q.

vector @ Q (+.5) direction (+.5) towards steepest increase

(d) [2] (Quiz6 #2) Let $\overrightarrow{u} = \langle \frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}} \rangle$. Find a point (c, d) where $D_{\overrightarrow{u}}(c, d) > 0$. Provide some brief justification.

(+.5) $\vec{u}$ $\swarrow$

any point above where when we travel down & to the left our outputs increase

(+.5) notation

(+1) eg P, R, A, C

5. The wind-chill index W is the perceived temperature when the actual temperature is T and the wind speed is v, so we can write $W = f(T, v)$. The table below provides some values for W as a function of T and v.

Wind speed (km/h)

Actual temperature (°C) T \ v	20	30	40	50	60	70
−10	−18	−20	−21	−22	−23	−23
−15	−24	−26	−27	−29	−30	−30
−20	−30	−33	−34	−35	−36	−37
−25	−37	−39	−41	−42	−43	−44

(a) [3] (§14.3 #2) Approximate $f_v(-15, 60)$ and interpret it for a 10 year old.

$\approx \frac{-30 - -30}{60-70} = 0 \ \frac{°C}{km/hr}$

or

$\approx \frac{-29 - -30}{50-60} = \frac{-1}{10} \ \frac{°C}{km/hr}$

(+.5)

graph reading (+.5) → When the wind is blowing 60 mi/hr if it blows harder the wind-chill (temperature we feel) will feel the same (assuming actual temp stays the same)

vary v (+.5)

+1.5

(b) [4] (Quiz6 #1) Find one local linearization for f that could be used to approximate both $W(-16, 58)$ or $W(-16, 61)$. Provide the *function*, not the actual approximations. (+.5) Looking for $W - W_0 = m_T(T - T_0) + m_v(v - v_0)$

(+.5) Use $(-15, 60)$ b/c close to $(-16, 58)$ and $(-16, 61)$

(+.5) $m_v = f_v(-15, 60) \approx 0$ from (a)

(+1) $m_T = f_T(-15, 60) \approx \frac{-36 - -30}{-20 - -15} = \frac{-6}{-5}$

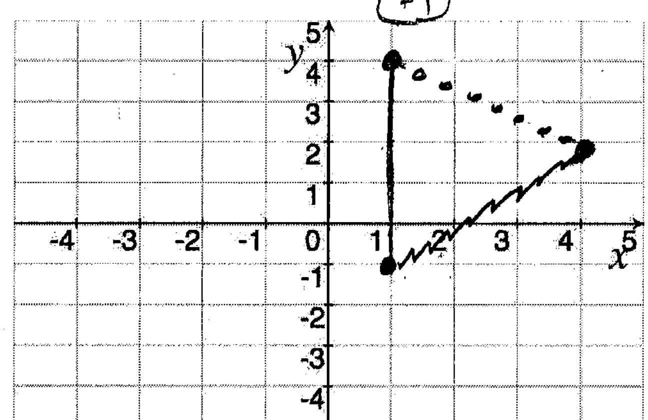
so (+.5)

$W - -30 = \frac{6}{5}(T - -15) + 0(v - 60)$

(+1)

6. Let the triangular region with vertices $(1, 4)$, $(4, 2)$ and $(1, -1)$ be denoted as D.

(a) [1] (§15.2 #4) Sketch the region D.



(b) [3] Set up the definite integral that is equivalent to the signed volume trapped by the function $z = 3x^2$ and above the region D. You do *NOT* need to compute this.

need the equation of the lines

(+1)

$m = \frac{-2}{3}$ thru $(1, 4)$

$y - 4 = -\frac{2}{3}(x - 1)$

$\Rightarrow y = -\frac{2}{3}x + \frac{2}{3} + 4$

$y = -\frac{2}{3}x + \frac{14}{3}$

$m = \frac{3}{3} = 1$

thru $(1, -1)$

$y + 1 = 1(x - 1)$

$y = x - 1 - 1$

$y = x - 2$

fix x & vary y

$\int^4 \int 3x^2 \, dy \, dx$ (+.5)

(+.5) (+.5)

$\int_1^4 \int_{x-2}^{-\frac{2}{3}x + \frac{14}{3}} 3x^2 \, dy \, dx$ (+.5)

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7. [6] For the following problem you will outline (not actually find!) a solution. Make sure your outline includes:

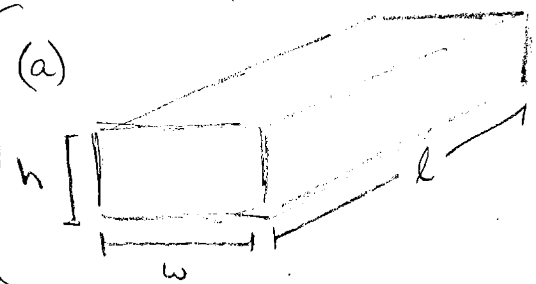
   (a) definitions of variables used,
   (b) identifying the function that needs to be optimized,
   (c) boxing systems of equations that need to be solved (but do not solve them!), &
   (d) explaining how you would verify your work is correct (ie a maximum)

§14.8 #54

start (+.5)

(§14.8 #) A package in the shape of a rectangular box can be mailed by the postal service if the sum of its length and girth (the perimeter of a cross-section perpendicular to the length) is at most 108 inches. We would like to find the dimensions of the package with the largest volume that can be mailed.

(+1) (a)



$l + \text{girth} \le 108$

$l + 2w + 2h \le 108$ unpacked girth (+.5)

$\text{Volume} = l \cdot w \cdot h$ (+.5)

(+1) (b) optimize $\text{Volume} = l \cdot w \cdot h$

subject to constraint:

$g(l,w,h) = l + 2w + 2h \le 108$

note: three variables/inputs?

OR $l = 108 - 2w - 2h$

$\Rightarrow \text{Vol} = (108 - 2w - 2h)\, w \cdot h$

$= 108wh - 2w^2h - 2wh^2$

only two inputs :)

(c) lets use Lagrange multipliers

partials (+1)
complete system (+.5)

$$\begin{cases} \dfrac{\partial V}{\partial l} = \lambda \dfrac{\partial g}{\partial l} \\ \dfrac{\partial V}{\partial w} = \lambda \dfrac{\partial g}{\partial w} \\ \dfrac{\partial V}{\partial h} = \lambda \dfrac{\partial g}{\partial h} \\ g(l,w,h) = 108 \end{cases} \Rightarrow \boxed{\begin{aligned} wh &= \lambda(1) \\ lh &= \lambda 2 \\ lw &= \lambda 2 \\ l + 2w + 2h &= 108 \end{aligned}}$$

OR looking for critical points

$$\begin{cases} \dfrac{\partial \text{Vol}}{\partial w} = 0 \\ \dfrac{\partial \text{Vol}}{\partial h} = 0 \end{cases} \Rightarrow \boxed{\begin{aligned} 108h - 4wh - 2h^2 &= \\ 108w - 2w^2 - 4wh &= \end{aligned}}$$

(+1) (d) Too many variables to use 2nd der. test :(

Once we get a critical point for l,w,h we need to put in a few values nearby into Vol. eq. ie fill out:

l	w	h	Vol
$l_0 \pm .1$	$w_0 \pm .01$	$h_0 \pm .08$	

← make sure the volume resulting is less

OR we can use the 2nd derivative test for this one :)

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