Key

Exam 1 # TMath 126 Autumn 2025

As a reminder, you are welcome to use a non-internet accessing calculator (which includes Desmos Test Mode) but no books, other notes, or peers. Show your work for the following problems. The correct answer with no supporting work will receive NO credit.

1. [6] TRUE/FALSE: Write True in each of the following cases if the statement is *always* true and provide a brief justification. Otherwise, write False and provide a counterexample or brief justification.

   (a) (3DActivity#2) A vector values function that describes the intersection of the paraboloid $z = 7x^2 + y^2$ and the parabolic cylinder $y = 3x^2$ is $\langle t, 3t^2, 7t^2 + 9t^4 \rangle$.

   stat (+.5)
   create parameter (+1)
   sub (+.5)
   justify (+.5)

   $z = 7x^2 + (3x^2)^2$

   $z = 7x^2 + 9x^4$

   if $x = t$, $y = 3t^2$

   so $\langle t, 3t^2, 7t^2 + 9t^4 \rangle$

   (+.5) True

   (b) (§13.2#26) If $\vec{r}(t) = \langle \ln(t+1), t\cos(2t), 2^t \rangle$, then the line tangent to $\vec{r}(0)$ is:

   $$\langle x, y, z \rangle = \langle 0, 0, 1 \rangle + s\langle \frac{1}{t+1}, -2t\sin(2t) + \cos(2t), 2^t(\ln 2) \rangle$$

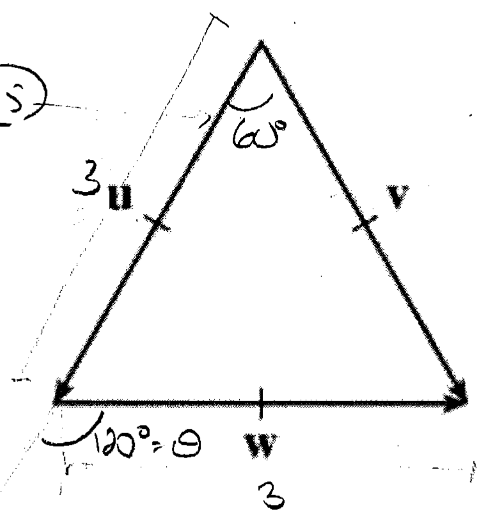
   ↳ not A directional vector

   ⇒ this is not a line

   stat (+.5)
   derivatives/eq of line (+1.5)
   justify (+.5)

   (+.5) False

2. Let $\vec{u}$, $\vec{v}$, and $\vec{w}$ be the vectors shown on the right with $3 = ||\vec{u}|| = ||\vec{v}|| = ||\vec{w}||$.



   (a) [2] (WebHW12.3#5) Find $\vec{u} \cdot \vec{w}$

   $||\vec{u}|| \cdot ||\vec{w}|| \cos\theta$ (+.5)

   $3 \cdot 3 \cos 120°$ (+1)

   $3 \cdot 3 \cdot -\frac{1}{2} = -\frac{9}{2}$

   (b) [2] (Quiz2#3) Find $\vec{u} \times \vec{w}$

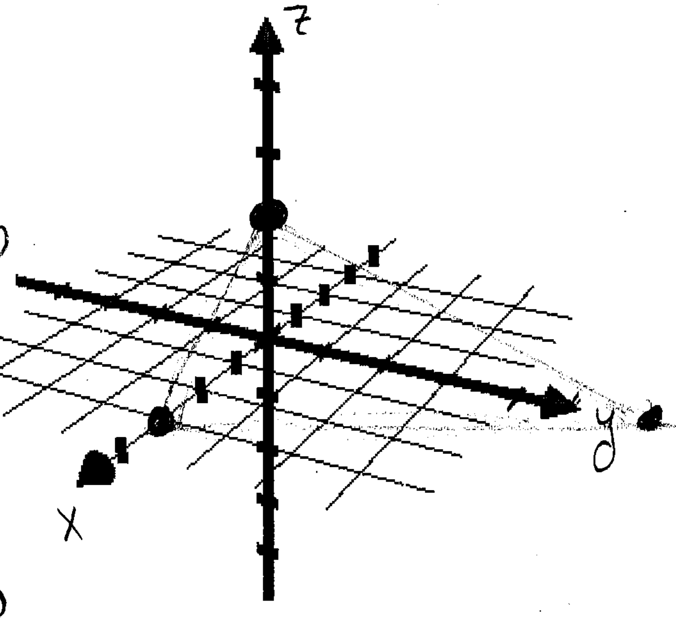
   $||\vec{u} \times \vec{w}|| = ||\vec{u}|| \, ||\vec{w}|| \sin\theta$ (+.5)

   $= 3 \cdot 3 \sin 120$ (+.5) $\vec{u}$

   $= 9\frac{\sqrt{3}}{2} \approx 7.8$ ↳ consistent with (c)

   out of the page (+.5) is a vector (+.5)

3. [3] (VectorActivity#1) Label the axis, plot $4x + 2y + 6z = 12$, and clearly label one point.



Axis (+.5) start (+.5)

note if $x=0$, $y=0$, $0+0+6z=12$

$\Rightarrow z=2$

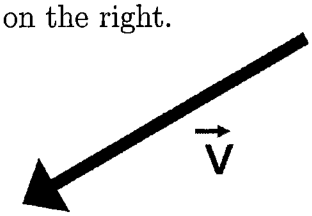
(+.5) so $(0,0,2)$ plot (+.5)

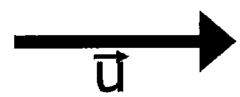
Similarly $(0,6,0)$ and $(3,0,0)$

looks like a plane shape (+.5)

gut it (+.5)

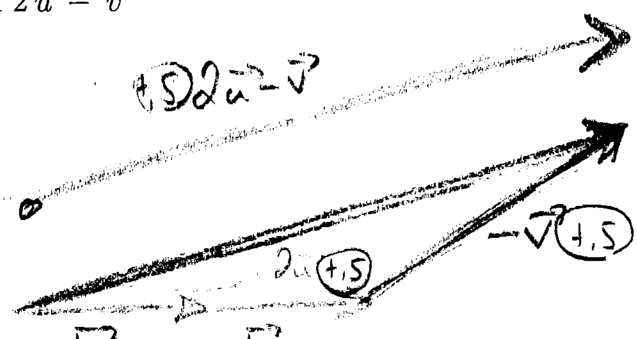
4. [3] (WrittenHW12.2 #6) Let $\vec{u}$ and $\vec{v}$ be the vectors shown on the right. Sketch $2\vec{u} - \vec{v}$





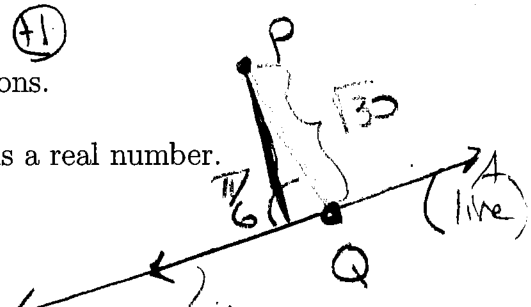
notation (+.5)

so



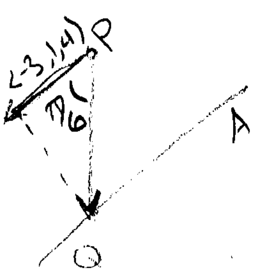
tip to tail (+1)

5. Use the following information below for the following questions.



- The object A is given by $\langle 1, 2, 0\rangle + \langle -3, 1, 4\rangle t$ where t is a real number.
- Let the point $Q \in \mathbb{R}^3$ satisfy the equation of A.
- Let P be another point such that $||\overrightarrow{PQ}|| = \sqrt{30}$,
- the angle between $\langle -3, 1, 4\rangle$ and $\overrightarrow{PQ}$ is $\frac{\pi}{6}$ radians.

(a) [1] (Quiz2#2) Identify if A is a line, a plane, or some other 3D object.



(+1) line

(b) [3] (Lines&PlanesActivity#4) Find the distance between the point P and A. Consider drawing a picture to more easily show your work.

Picture (+1) (+.5) Sohcahtoa

(+1) $\sin \pi/6 = \dfrac{\text{distance wanted}}{||\overrightarrow{PQ}||}$

$\Rightarrow$ distance $= \sqrt{30}\,\sin \pi/6$ (+.5)

≈ 2.738

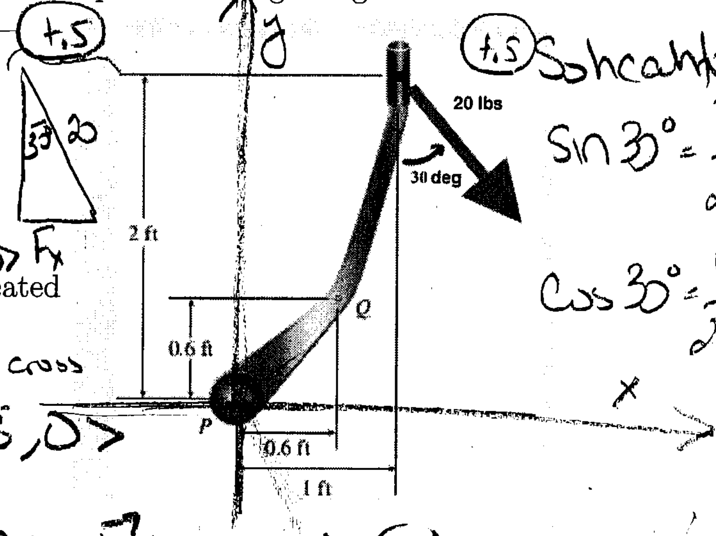
$= \sqrt{30} \cdot \frac{1}{2} = \frac{\sqrt{30}}{2}$

6. (WrittenHW12.4 #40) Consider the bicycle pedal shown on the right. A force of 20 lbs is applied at an angle as shown in the same plane as the gearing.

(a) [3] Draw a set of axis and write the components of the force vector with respect to your axis. (+1) (+.5) (+.5) (+.5)

$\langle 20\sin 30°, 20\cos 30°, 0\rangle$ F_y

$\langle 20\cdot\frac{1}{2}, 20\frac{\sqrt{3}}{2}, 0\rangle \approx \langle 10, 17.3, 0\rangle$ F_x



(+.5) Sohcahtoa

$\sin 30° = \frac{F_x}{20}$

$\cos 30° = \frac{F_y}{20}$

(b) [2] Find the *vector* of the torque created about the pivot point P.

$\vec{\tau} = \vec{r} \times \vec{F}$ (+.5) (+.5) def of cross

$= \langle 1, 2, 0\rangle \times \langle 10, -10\sqrt{3}, 0\rangle$ (+.5)

$= (-10\sqrt{3} - 20)\vec{k} \approx -37.3\vec{k}$ vector (+.5) so into page

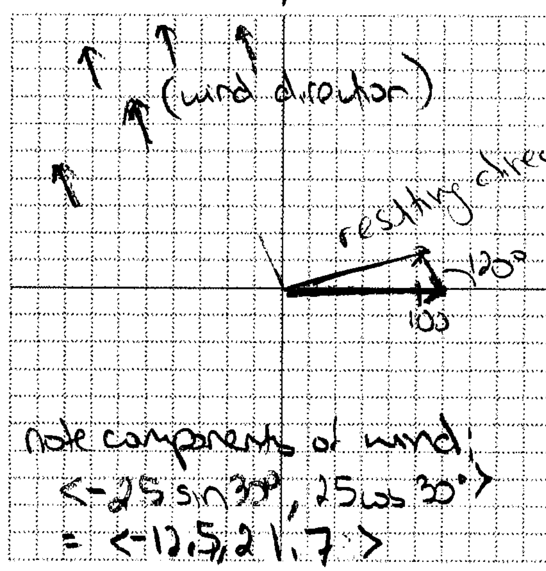
7. (WordProblem #2) A plane is flying at 120 knots and there is a wind with a speed of 25 knots in the direction of 120° from due east.

(a) [2] If the pilot steers the plane directly due east, approximately what direction will the plane flight take? Briefly justify your answer.

start (+.5)
(+.5) picture

(+.5) [east but a bit north (purple vector)

(+.5) [this is vector addition of the velocity vectors.



(b) [3] What angle from due east does the pilot need to steer the plane so that the resulting flight path is due east?

(+.5) The pilot will need to adjust slightly to the south so the vector sum lands on the east like

airspeed wind E

(+.5) Let $\vec{w}$ be the wind speed vector & $\vec{a}$ be airplane vector

(+.5) want $w_y + a_y = 0$

$\|\vec{w}\|\cos 30° + \|\vec{a}\|\sin\theta = 0$

$25\cos 30 + 120\sin\theta = 0$

$\Rightarrow \sin\theta = (-25\cos 30°)/120°$

$\Rightarrow \theta \approx \arcsin(-.18) \approx -10.4$ degrees (+.5)

(+1)

Sohcahtoa

$\vec{w}$ w_y 120°

$\cos 30° = \frac{w_y}{\|\vec{w}\|}$

θ $\vec{a}$ a_y

$\sin\theta = \frac{a_y}{\|\vec{a}\|}$

8. [4] (WebHW10.2#3) Given the following information, find the line tangent to the curve $x = f(t)$, $y = g(t)$ when $t = \frac{1}{2}$. Use whatever form of a line you like (eg. parametric, slope-intercept, standard, etc)

Start +.5
Notation +.5

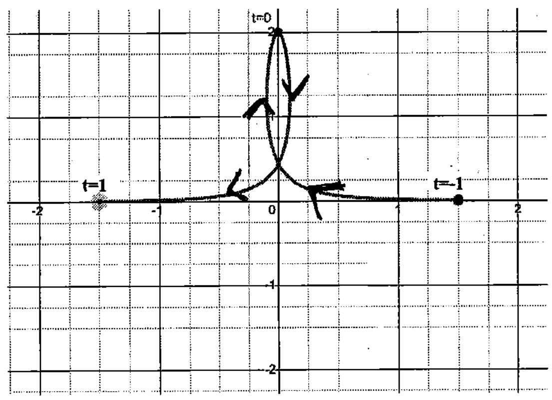
$f\left(\frac{1}{2}\right) = 3.2$ $g\left(\frac{1}{2}\right) = -1.5$ $\frac{df}{dt}\left(\frac{1}{2}\right) = 2.8$ $\frac{dg}{dt}\left(\frac{1}{2}\right) = 1$

vector form:
$\langle x_0, y_0 \rangle + t \langle v_1, v_2 \rangle$ +1
$\langle x, y \rangle = \langle 3.2, -1.5 \rangle + t \langle 2.8, 1 \rangle$ +1 +1 $t \in \mathbb{R}$

OR precalc form:
$y - y_1 = m(x - x_1)$ +1
$m = \left.\frac{dy}{dx}\right|_{t=1/2} = \frac{\left.dy/dt\right|_{t=1/2}}{\left.dx/dt\right|_{t=1/2}} = \frac{1}{2.8}$ +1

So $y + 1.5 = \frac{1}{2.8}(x - 3.2)$ +1

9. (PracticeExam1#4) Consider the parametric curve $x = f(t)$, $y = g(t)$ where $-1 \le t \le 1$, graphed below for the following questions.



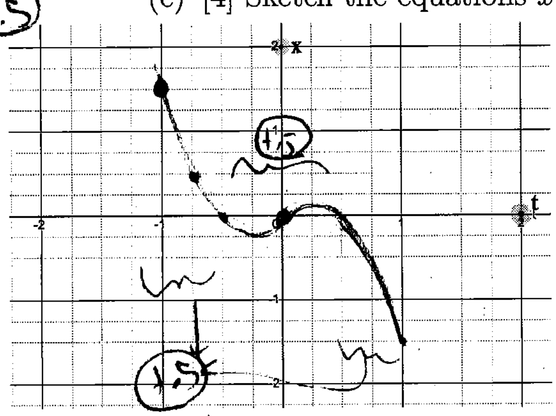
(a) [1] (WebHW13.1#1) Note the t values recorded on the graph. Indicate the direction of which t increases with an arrow on the graph.

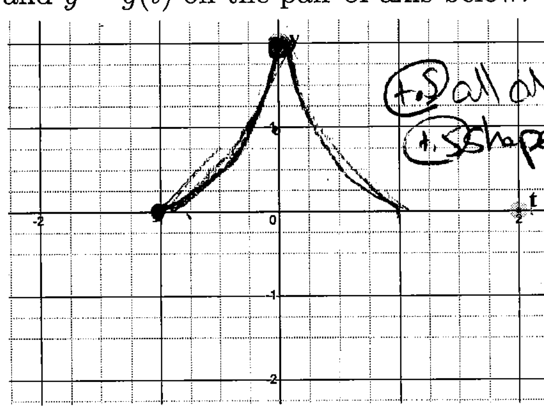
(b) [1] Looking at the graph, approximate where $\frac{dy}{dx}$ is not defined. (Report either a point on the graph or an approximate t value.)

- end points $t = 1$ and -1
- vertical tangent lines $\approx (.1, 1)$ and $(-.1, 1)$
- where crosses self @ $\approx (0, .3)$

(c) [4] Sketch the equations $x = f(t)$ and $y = g(t)$ on the pair of axis below.

Start +.5





+.5 all along t axis
+.5 shape

t	x	y
-1	1.5	0
-.75	.5	.1
-.5	0	.5
-.25	-.1	1
0	0	2

} graph reading +1.5