The class before the exam there will be a chance to earn extra credit. Groups of three to four can present a solution to one of the problems below. Up to 4% can be earned:

- [1] Mastery of the problem: Do you understand the problem and all of the steps used to solve it? Would you be able to solve the problem if given a slightly different question?
- [1] Presentation of the problem: You are presenting material to your classmates that will be on their exam next week! Take care to explain your steps and why you take them but your group also needs to complete your presentation in under 10 minutes!
- [1] Presentation: Do you interact with your audience? Do you make eye contact?
- [1] Fielding questions: Can you understand the questions and give a cohesive answer?

Word Problem Practice take 3

1. One method to slow the growth of an insect population without using pesticides is to introduce a number of sterile males that mate with fertile females but produce no offspring. If P represents the number of female insects in a population, S the number of sterile males introduced each generation, and r the population's natural growth rate, then the female population is related to time t by

   $$t = \int \frac{P+S}{P[(r-1)P-S]}\, dP.$$

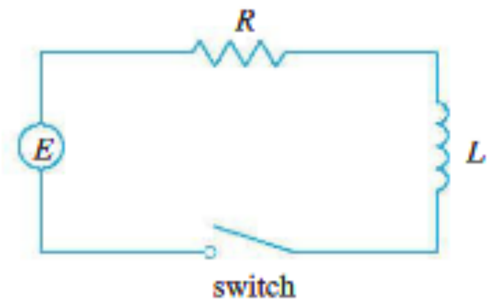
   Suppose a population with 10,000 females grows at a rate of $r = 0.10$ and 900 sterile males are added in each generation. Evaluate the integral to give an equation relating the female population to time. Note, you do not need to explicitly solve for P.

2. Newton's Law of Cooling states that the rate of cooling of an object is proportional to the temperature difference between the object and its surroundings, provided that this difference is not too large. Write down the differential equation just described. Use this differential equation to find a function $H(t)$ that obeys Newton's Law of Cooling where $H(t)$ gives the current temperature of a freshly poured cup of coffee (with temperature 190°F) in a room where the temperature is 70°F. You should have trouble finding the function H, what other information do you need to find H exactly?

3. Psychologists interested in learning theory study learning curves. A learning curve is the graph of a function $P(t)$, the performance of someone learning skill as a function of the training time t. The derivative dP/dt represents the rate at which performances improves & is believed to be proportional to the difference between the maximum level of performance of which the learner is capable (call it M) and the actual performance (P). Create and then solve a differential equation involving the function P.

4. Newton's Law of Gravitation states that two bodies with masses m_1 and m_2 attract each other with a force $F = G\frac{m_1 m_2}{r^2}$ where r is the distance between the bodies and G is the gravitational constant. Compute the work required to launch a 1000.0 kg satellite out of earth's gravitational field. Assume the earth's mass is 5.98×10^{24}kg and is concentrated at its center. The radius of the earth is about 6.37×10^{6} m and let $G = 6.67 \times 10^{-11}$ Nm^2/kg^2.

5. Consider the electric circuit shown to the right. The circle is a battery supplying a constant 60 volts (V) and a current I amperes (A) at time t. The circuit also contains a resistor with resistance 12 ohms (Ω) and an inductor with inductance 4 henries (H).



   Ohm's Law gives the drop in voltage due to the resistor as RI. The voltage drop due to the inductor is $L(dI/dt)$. One of Kirchhoff's laws says that the sum of the voltage drop is equal to the supplied voltage $E(t)$.

   Piece the above physics laws together to form a differential equation

6. Experiments show that the reaction $H_2 + Br_2 \to 2HBr$ satisfies the rate law

$$\frac{d[\text{HBr}]}{dt} = k[\text{H}][\text{Br}]^{\frac{1}{2}}$$

   where [H] is the original concentrations of a moles of H per L,
   [Br] is the original concentrations of b moles of Br per L, and
   [HBr] is the concentrations of x moles of HBr per L. Explain why the above differential equation can be rewritten as

$$\frac{dx}{dt} = k(a-x)(b-x)^{\frac{1}{2}}.$$

   Find x as a function of t in the case where $a = b$ and assume $x(0) = 0$.

7. A glucose solution is administered intravenously into the bloodstream at a constant rate r. As the glucose is added, it is converted into other substances and removed from the bloodstream at a rate that is proportional to the concentration at that time. Let $C(t)$ be the concentration of glucose solution in the bloodstream and assume $C(0) = 0$. Find the rule of $C(t)$ by solving a differential equation and describe the behavior as $t \to \infty$.

8. Justify each of the factors in the logistic differential equation as a model for population growth:

$$\frac{dP}{dt} = kP\left(1 - \frac{P}{M}\right)$$

   Be sure to explain what M is. Use the logistic model to find the population P of fish in a lake at time t. Biologists first stocked the late with 400 fish and estimated the carrying capacity to be 10,000. Biologists returned in a year and found that the population had already tripled.

9. Consider the region trapped between $f(x) = \frac{1}{x}$, the x-axis, and from $x = 0$ to $x = 1$. What is the area of the region? What would it's volume be if it was revolved about the x-axis? What would it's volume be if it was revolved about the y-axis?

10. Household electricity is supplied in the form of alternating current that varies from 155V to -155V with a freqency of 60 cycles per second (Hz). The voltages is thus given by the equation

$$E(t) = 155 \sin(120\pi t)$$

where t is the time in seconds. Voltmeters read the RMS (root-mean-square) voltage, which is the square root of the average value of $[E(t)]^2$ over one cycle.

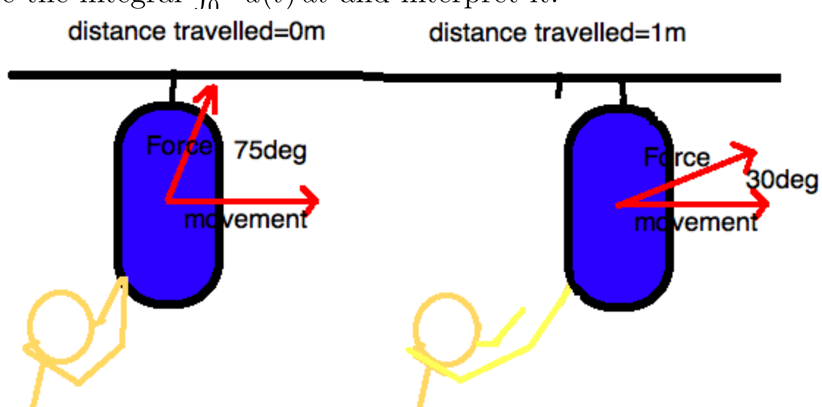
(a) Calculate the RMS voltage of household current.

(b) Many electric stoves require an RMS voltage of 220V. Find the corresponding amplitude A needed for the voltage $E(t) = A \sin(120\pi t)$.

11. Dialysis treatment removes urea and other waste products from a patient's blood by diverting some of the bloodflow externally through a machine called a dialyzer. The rate at which the urea is removed from the blood (in mg/min) is often well described by the equation

$$u(t) = \frac{r}{V} C_0 e^{\frac{-rt}{v}}$$

where r is the rate of flow of blood through the dialyzer (in mL/min), V is the voluem of the paient's blood (in mL), and C_0 is the amount of urea in the blood (in mg) at time $t = 0$. Evaluate the integral $\int_0^\infty u(t)\,dt$ and interpret it.



12. A factory worker is trying to push a large package suspended from a track on the ceiling a meter to the right. Conveniently the worker's arm length is 1 meter and she can apply 130 Newtons to do so. However, given her short height she can only apply the force at an angle. Initially she can only push the package up and to the right making an angle of 75° with the horizontal, but by the end of the 1 meter she has a better angle of 30° (picture attempted below). Assume the angle varies linearly with the distance that the package travels. How much work does the factory worker do on the object?