Differential Equation Examples

worksheet based on material provided by Dr. Eaton

1. Dr. Whodunit is found dead in his office at 5:00pm one evening. The temperature of his body was 80.0°F. One hour later, at 6:00pm, the body has cooled to 75.0°F. The room is kept at a constant temperature of 70°F. Assume Dr. Whodunit had a normal temperature of 98.6°F at the time of death.

   Let $f(t)$ be the temperature of the body after t hours.

   (a) Since the rate a body cools is proportional to the difference in temperature between the body and the ambient temperature, we can write down a differential equation. Recall "proportional to" means "is a constant k times".

   (b) The formula you wrote down above is known as Newton's Law of Cooling and happens to be a separable differential equation. Use the techniques we covered in class today to solve the differential equation you wrote in part (a). Treat k as a constant and leave the "$+c$" on the side with the t's.

   (c) Solve for $f(t)$ as a function of t. Note, your function will involve the constants k and c.

   (d) Recall that you were given two data points in the above information. Let $t = 0$ correspond to the time the body was found and solve the initial value problem and give us an explicit function $f(t)$ that models the temperature of Dr. Whodunit's body.

   (e) When did the murder take place?

2. The University of Washington, Tacoma has 3000 students. On Monday two students noticed police outside Dr. Whodunit's office and heard Dr. Whodunit was dead. A rumor began to spread and by Tuesday 200 students had heard it. It is reasonable to assume that rate of the spread of the rumor is proportional to the number of possible encounters between students who have heard the rumor and those who have not. Let $y = y(t)$ be the number of students who have heard the rumor after t days.

   (a) Explain why the possible meetings between students who have heard the rumor and those who have not equals $y(3000 - y)$.

   (b) Write a differential equation that describes this model of the spread of a rumor.

   (c) Solve the differential equation for $y(t)$. Note that your answer should involve two constants much like the previous problem did.

   (d) Recall that you were given two data points in the above information. Solve the initial value problem and give us an explicit function $y(t)$ that models the spread of the rumor.

   (e) When will 2000 people have heard the rumor?