## Integrating Rational Functions

While working in a group make sure you:

- Expect to make mistakes but be sure to reflect/learn from them!
- Are civil and are aware of your impact on others.
- Assume and engage with the strongest argument while assuming best intent.

1. $\int \frac{\frac{-1}{10}}{x+2} d x$
$\int \frac{\frac{1}{5}}{2 x-1} d x$
2. Generalize the above examples, that is, describe your strategy for integrating rational functions of the form $\frac{a}{b x+c}$ where $a, b$, and $c$ are constants.
3. $\int \frac{x}{x^{2}+2} d x$
$\int \frac{4 x}{5 x^{2}+7} d x$
4. Generalize the above examples, that is, describe your strategy for integrating rational functions of the form $\frac{a x}{b x^{2}+c}$ where $a, b$, and $c$ are constants.

Important derivative!!! (my favorite function!!)

$$
\text { 5. } \int \frac{1}{x^{2}+4} d x
$$

$$
\begin{aligned}
& \frac{d}{d u} \arctan (u)=\frac{1}{u^{2}+1} \\
& \qquad \int \frac{2}{9 x^{2}+1} d x
\end{aligned}
$$

For each factor of the form $(p x+q)^{m}$, the partial fraction decomposition must include the following sum of $m$ fractions:

$$
\frac{A_{1}}{p x+q}+\frac{A_{2}}{(p x+q)^{2}}+\frac{A_{3}}{(p x+q)^{3}}+\ldots \frac{A_{m}}{(p x+q)^{m}}
$$

For each factor of the form $\left(a x^{2}+b x+c\right)^{n}$, the partial fraction decomposition must include the following sum of $m$ fractions:

$$
\frac{B_{1} x+C_{1}}{a x^{2}+b x+c}+\frac{B_{2} x+C_{2}}{\left(a x^{2}+b x+c\right)^{2}}+\frac{B_{3} x+C_{3}}{\left(a x^{2}+b x+c\right)^{3}}+\ldots \frac{B_{n} x+C_{n}}{\left(a x^{2}+b x+c\right)^{n}}
$$

Example: $\int \frac{x^{3}}{x^{2}+4 x+3} d x$

