FINAL TMath 125 Autumn 2026

Reasonable supporting work must be shown to earn credit.

1. Each of the following is wrong. Explain why.

   (a) [2] (Exam1#6) Let $u = \cos(x)$, then $\displaystyle\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{\sin(x)}{\cos^2(x)}\,dx = \int_{\frac{\sqrt{3}}{2}}^{\frac{1}{2}} \frac{1}{u^2}\,du = -u^{-1}\big]_{\frac{\sqrt{3}}{2}}^{\frac{1}{2}}$

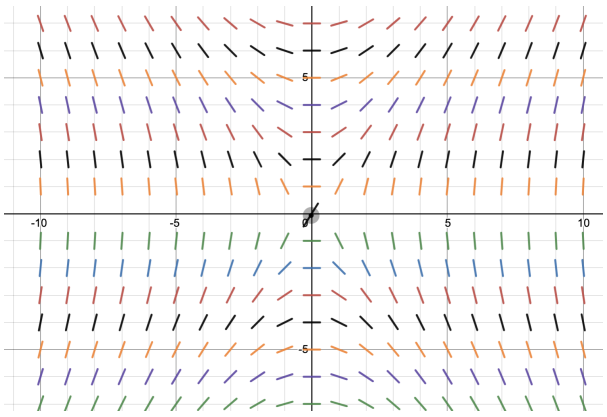
   (b) [2] (Written7-3 #40) During a trig. substitutions we let $3x = 6\sec(\theta)$. This substitutions allows us to write $12\int \tan^2(\theta)\sec(\theta)\,d\theta$ as $12\displaystyle\int \left(\frac{\sqrt{9x^2-36}}{6}\right)^2 \left(\frac{x}{2}\right)\,dx$

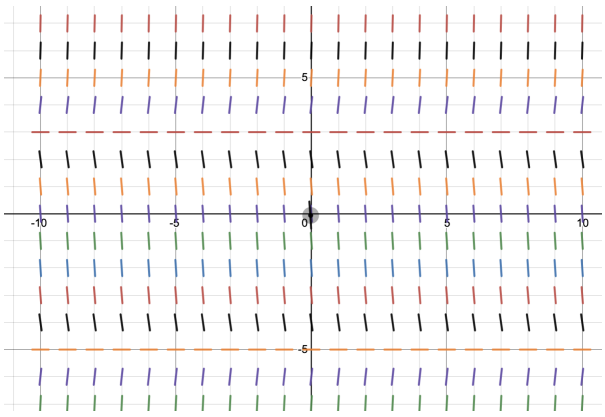
   (c) [2] (Exam2#4) $\displaystyle\int xe^x\,dx = \frac{1}{2}x^2e^x + C$

   (d) [2] (WebHW7-8#2) The signed area trapped by x^{-1}, the x-axis, and to the right of $x = 1$, is finite.

2. [3] (SummationActivity #1) Expand $\sum_{i=0}^{3} \left(\frac{(-1)^i}{i+3} \right)$.
(You do *not* need to compute or simplify this!)

3. [3] (Quiz9#2) Identify which of the following slope fields correspond to the differential equation: $\frac{dy}{dx} = \frac{2x}{y}$. Justify your answer.



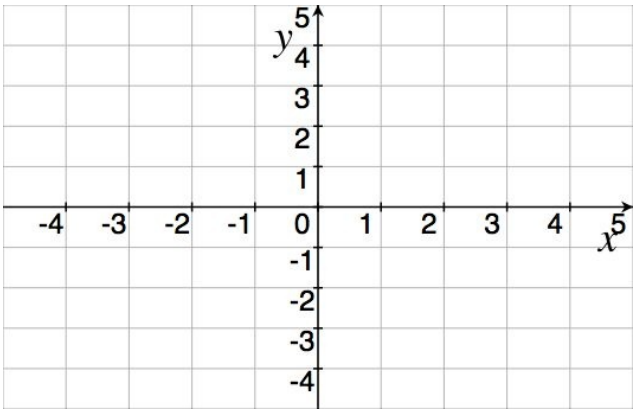


4. Oil leaked from a tank at a rate of $r(t)$ liters per hour. The rate decreased as time passed, and values of the rate at two-hour time intervals are shown in the table.

t	0	2	4
r(t)	4.4	3.9	3.5

(a) [1] Plot the points on the axes provided.

(b) [3] (WebHW5-1#4) Find a lower estimate for the total amount of oil that has leaked in the 4 hours.



(c) [2] (WrittenHW6.5 #16) Estimate the average rate the oil is spilling in the first 4 hours.

5. (Exam2 #2) For each of the following, identify the technique you would use to find the indefinite integral. For example,

   - if you think substitution, write "substitution" and identify u and du.
   - if you think integration by parts, write "by parts" and identify u, dv, du and v.
   - if you think trigonometric substitution, write "trig. sub." and identify θ and $d\theta$.
   - if you think partial fractions, write "partial fractions" and identify the form of the partial fraction.
   - if you think there is a different technique, identify it and set up the technique.

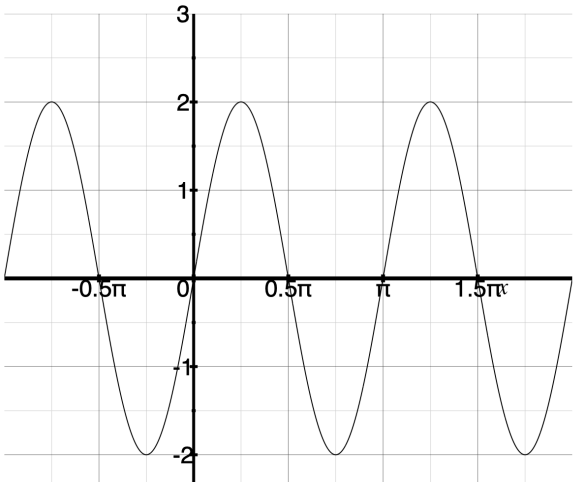
   (a) [2]
   $$\int \frac{2}{1+9x^2}\,dx$$

(b) [2]
$$\int \frac{2t-1}{t^2-3t+2}\,dt$$

(c) [2]
$$\int y2^y\,dy$$

6. [4] Evaluate *one* of the indefinite integrals above. Be careful of notation.

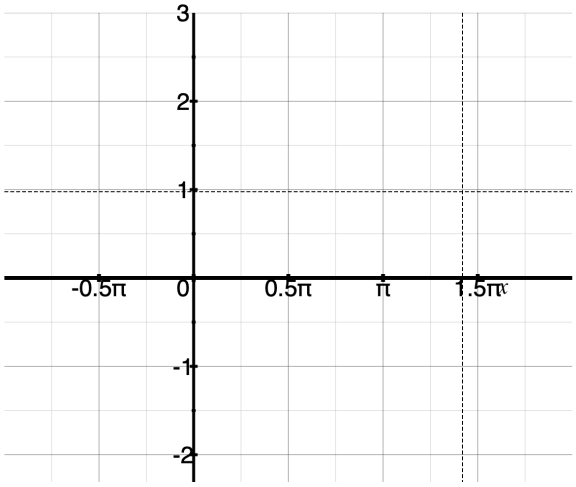
7. Consider a particle whose velocity, v, is given by $4\sin(t)\cos(t)$ feet per second at time t. The graph is provided.



   (a) [1] (Exam2#8) Find when the particle is traveling approximately 1 foot/sec.

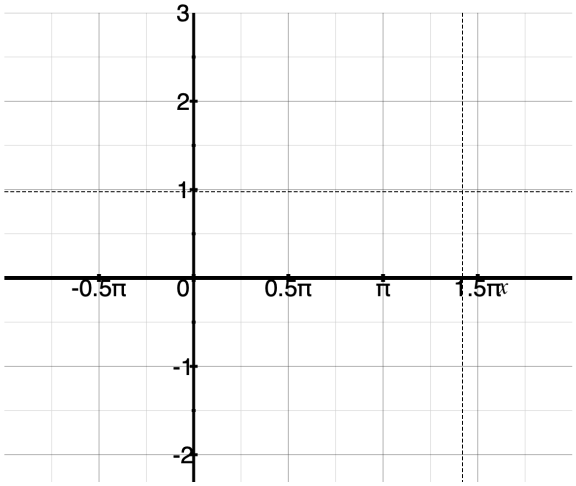
   (b) [1] (WrittenHW5-4#68) Identify a point in time when the particle is changing direction.

   (c) [2] (DefiniteIntActivity #2) Define $p(x) = \int_0^x v(t)\,dt$. Approximate $p(2)$.



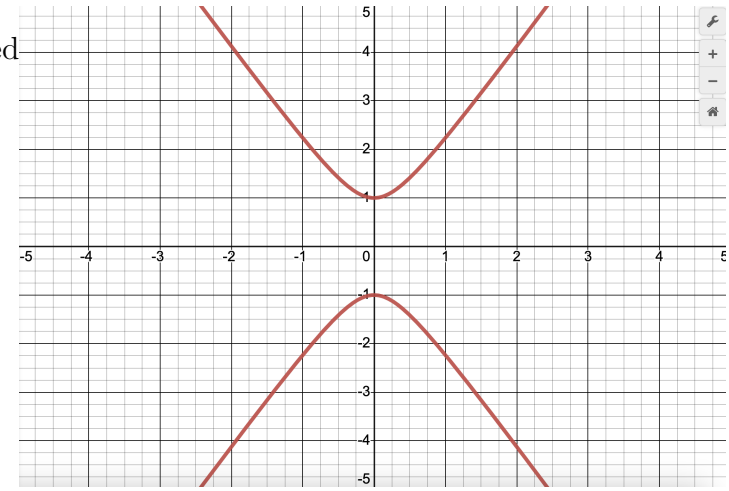
   (d) [4] (WebHW5-4#4) Sketch the graph of $p(x)$ if we know at time $t = 0$, the position is at 0.

   (e) [3] (WordProblem2#1) Find the acceleration of the particle when $t = 0$.



   (f) [2] (WrittenHW5.3 #4) Sketch the graph of $\frac{d}{dx}(p(x))$.

8. The graph of $y^2 - 4x^2 = 1$ is given below.



(a) [2] (Exam1#9) Shade the region trapped by $y^2 - 4x^2 = 1$, $x = 2$, and the y-axis.

(b) [4] (WrittenHW§7.3#40) Set up the definite integral for the area of the shaded region. Get your answer to the place that it can be given directly to Desmos.

(c) [2] (§6.2#2) Consider the volume whose base was shaded in (a). The cross sections (perpendicular to the x axis) are rectangles with a height, half that of the base. Sketch one of the approximating cylinders of this volume.

(d) [4] (WordProblem2#7) Set up the definite integral that would find the volume described above.

9. [3] (Exam2 #1) Given that $f(1) = 4$, $f(4) = 5$, $f'(1) = 7$, $f'(4) = 5$ and that f'' is continuous. Find $\int_1^4 2xf''(x)\,dx$.

10. [3] (WebHW9.3#7) A vat with 300 gallons of solution contains 4% vinegar (by volume). Another solution with 6% vinegar is pumped into the vat at a rate of 2 gal/min and the (well mixed) mixture is pumped out at the same rate. Write down the differential equation for the amount of vinegar in the vat at time t. Be sure to define any variables created.

11. [4] (Suggested 9.3 #13) Find an equation of the curve that passes through (0,0) and whose slope at (x, y) is xe^y.