Key

EXAM 2 TMath 125 Spring 2026

Show *all* your work. Reasonable supporting work must be shown for any partial credit.

1. Let $f(1) = 2$, $f(5) = -3$, $f'(1) = 4$, $f'(5) = 1$, and assume f'' is continuous.

   (a) [2] (PracticeExam2 #4) Evaluate $\int_1^5 6f''(x)\,dx$

   $= 6\int_1^5 f''(x)dx = 6f'(x)\Big]_1^5$ (+1) (+.5)

   $= 6(f'(5) - f'(1))$

   $= 6(1-4) = -18$ (+.5)

   (b) [3] (WebHW7.1 #10) Evaluate $\int_1^5 x f''(x)\,dx$

   $= xf'(x)\Big]_1^5 - \int_1^5 f'(x)\,dx$ (+.5)

   (+1) $u = x \quad v = f'(x)$; $du = dx \quad dv = f''(x)dx$

   $= \underbrace{5f'(5) - 1f'(1)}_{(+.5)} - f(x)\Big]_1^5$

   $= 5(1) - 1(4) - \underbrace{[-3-2]}_{(+.5)} = 1 + 5 = 6$ (+.5)

2. [4] (OldExam2 #3 & WebHW's) For each of the following, identify the technique you would use to find the indefinite integral. For example, if you think substitution would work, write "substitution" and identify what u would be. If you think integration by parts, write "integration by parts" and identify what u and du would be.

   sense/notation (+.5) (a) $\int \frac{x^3}{\sqrt{16+x^2}}\,dx$ $= \int \frac{x^2}{\sqrt{u}}\,x\,dx = \int \frac{u-16}{u^{1/2}}\left(\frac{1}{2}\right)du = \frac{1}{2}\int \frac{u}{u^{1/2}} - \frac{16}{u^{1/2}}\,du$

   (+1) $\rightarrow u = 16 + x^2$, $du = 2x\,dx$ $\Rightarrow x^2 = u - 16$ $\Rightarrow \frac{1}{2}du = x\,dx$

   $= \frac{1}{2}\int u^{1/2} - 16u^{-1/2}\,du = \frac{1}{2}\left[\frac{2}{3}u^{3/2} - 16\cdot 2u^{1/2}\right]$

   (+.5) method works

   note: trig should work too? $= \frac{1}{3}u^{3/2} - 16u^{1/2} + C = \frac{1}{3}(16+x^2)^{3/2} - 16(16+x^2)^{1/2} + C$

   (b) $\int \sin(5x)\sec(5x)\,dx$

   sense/notation (+.5) $= \int \sin(5x)\cdot\frac{1}{\cos(5x)}\,dx = \int \frac{\sin(5x)}{\cos(5x)}\,dx = \int \frac{1}{u}\left(\frac{-1}{5}\right)du = \frac{-1}{5}\int \frac{1}{u}\,du$

   $= -\frac{1}{5}\ln|u| + C$

   (+1) $\rightarrow u = \cos(5x)$

   method works (+.5) $du = -5\sin(5x)dx \Rightarrow -\frac{1}{5}du = \sin(5x)dx$

   $= -\frac{1}{5}\ln(\cos(5x)) + C$

3. [4] Evaluate *one* of the indefinite integrals above. Clearly indicate the work you would like considered.

   start (+.5)
   notation (+.5)
   tried method (+.5)
   did method correctly (+1.5)
   derivatives/integrals/identities/exp/alg (+1)

   note alternative to 2(a)
   use $\tan^2\theta + 1 = \sec^2\theta$
   $x = 4\tan\theta \quad dx = 4\sec^2\theta\,d\theta$
   $\int \frac{4^3\tan^3\theta}{\sqrt{16+16\tan^2\theta}}\cdot 4\sec^2\theta\,d\theta = \int \frac{4^3\tan^3\theta\sec^2\theta\,d\theta}{4\sec\theta}$
   $= 4^3\int \tan^3\theta\,d\theta\sec\theta\,d\theta$

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4. [4] Each of the following is wrong. Find the step with the error and explain why it is wrong.

(a) $\int xe^{9x}\,dx = \frac{1}{2}x^2 \cdot \frac{1}{9}e^{9x} + C$

only 1 place would be wrong ☺ (+.5)

(+1) [Looks like integrated the factors seperately but $\int f(x)g(x)dx \neq \int f(x)dx \int g(x)dx$

correct/notation (+.5)

(b) $\int \sqrt{16-x^2}\,dx = \int \sqrt{16-(4\sin(x))^2}\,dx = \int 4\cos(x)\,dx = 4\sin(x) + C$

(+.5) looks like trying to use trig substitution

correct/notation (+.5)

(use diff variable)

trouble:

either (+1) [
- using x in diff way $x = 4\sin\theta$
- need to swap out dx for $d\theta$

5. Consider the triangular base with the vertices: $(-1, 3)$, $(2, 1)$, and $(2, 4)$.

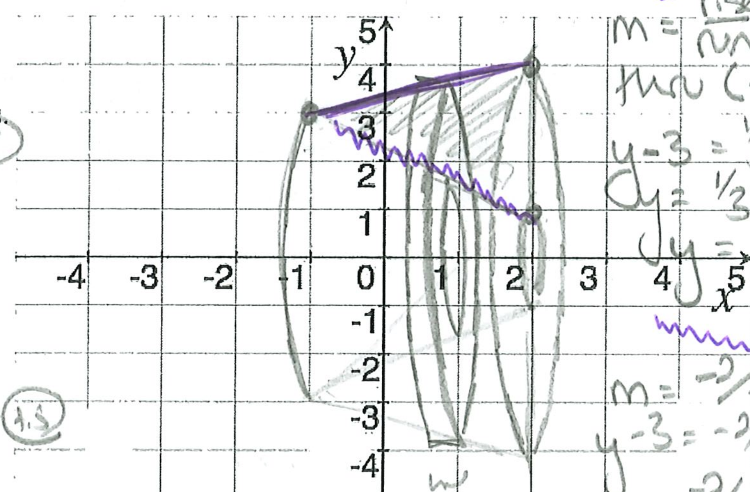
(a) [2] (WebHW6.2 #6)
Graph region on the axis.

got each point (+.5)
region (+.5)

(b) [2] (§6.2 #2)
If the region in (a) was revolved around the x-axis, sketch what a typical disk would look like.

got it (+.5)

shape (+1)
⊥ to x-axis (+.5)



$m = \frac{\text{rise}}{\text{run}} = \frac{1}{3}$ thru $(-1,3)$ so
$y - 3 = \frac{1}{3}(x - -1)$
$y = \frac{1}{3}x + \frac{1}{3} + 3$
$y = \frac{1}{3}x + \frac{10}{3}$

$m = -\frac{2}{3}$ thru $(-1,3)$
$y - 3 = -\frac{2}{3}(x - -1)$
$y = -\frac{2}{3}x - \frac{2}{3} + 3$
$y = -\frac{2}{3}x + \frac{7}{3}$

(c) [4] (WebHW5.4&5.3 #12) Set up the definite integral that would find the volume if the region in (a) was revolved around the x-axis.

Add up Δx = Add (outside) − (hole) (+.5) = Add up $\pi R^2 \Delta x - \pi r^2 \Delta x$ (+1) (+.5)

big radius   hole radius

$= \pi(\text{y value of — line})^2 \Delta x - \pi(\text{y value of} \sim\sim \text{line})^2 \Delta x$

$= \int_{-1}^{2} \pi\left(\frac{1}{3}x + \frac{10}{3}\right)^2 dx - \int_{-1}^{2} \pi\left(-\frac{2}{3}x + \frac{7}{3}\right)^2 dx$ (+.5) (+.5)

end points (+1)

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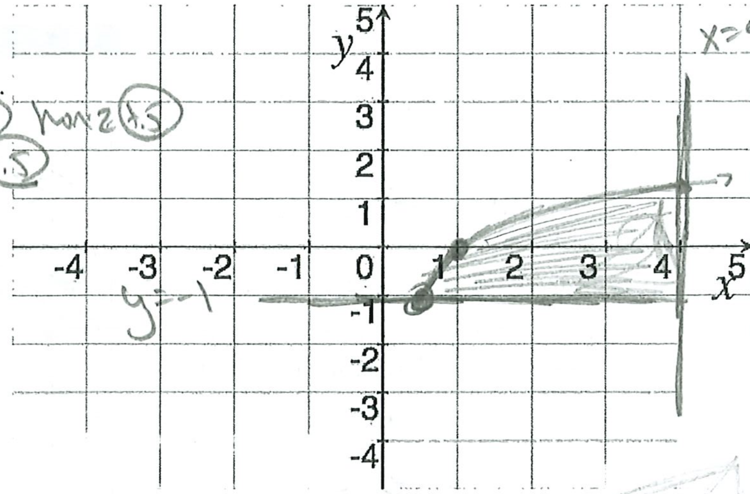
6. Consider the area trapped between $\ln(x)$, $y = -1$, and $x = 4$.

(a) [2] (WebHW6.2 #1) Graph region on the axis.

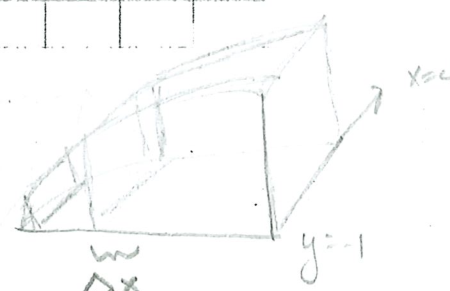


$y = \ln(x)$ (+.5) vert (+.5) horiz (+.5) region (+.5)

(b) [4] (WordProblem #7) If the region in (a) was used as a base for a volume whose cross sections formed squares when cut perpendicular to the x axis. Set up the definite integral that would find the volume.

$-1 = \ln(x)$

$e^{-1} = x$



sum up $(\text{side})^2 \cdot \Delta x$ (+.5) (+.5) (+.5)

limits (+1)

$$\int_{1/e}^{4} (\text{side})^2\,dx = \int (\text{dist } y \text{ coord to } y=-1)^2\,dx$$

$$= \int_{1/e}^{4} (\ln(x) - -1)^2\,dx = \int_{1/e}^{4} (\ln x + 1)^2\,dx$$ (+1)

start / cross sections / drew (+.5)

note $1/e \approx .36788$

7. One problem made the substitution $3x = 6\sec(\theta)$ and then integrated to get the answer $\frac{1}{2}\sec(\theta)\tan(\theta) - \frac{1}{2}\ln|\sec(\theta) + \tan(\theta)|$.

(a) [2] (TrigSub Activity #3) Find $\cos(\theta)$ as a function of x.

$$\frac{3x}{3} = \frac{6\sec(\theta)}{3}$$ (+.5)

$$x = 2\sec(\theta)$$

$$x = 2\frac{1}{\cos\theta}$$ (+.5)

fractions/alg (+1)

$$\Rightarrow x\cos\theta = 2$$

$$\cos\theta = \frac{2}{x}$$

got + (+.5)

def of $\sec\theta = \frac{1}{\cos\theta}$ (+.5)

algebra (+1)

(b) [3] (WebHW7.3 #2) Convert the answer (that is currently in terms of θ) back into terms of x.

Since $3x = 6\sec\theta \Rightarrow \sec\theta = \frac{3x}{6} = \frac{x}{2}$ (+.5)

Recall $\tan^2\theta + 1 = \sec^2\theta$ so $\tan^2\theta = \sec^2\theta - 1$ (+.5)

$$\Rightarrow \tan^2\theta = \left(\frac{x}{2}\right)^2 - 1 = \frac{x^2}{4} - 1 = \frac{x^2-4}{4}$$

$$\Rightarrow \tan\theta = \sqrt{\frac{x^2-4}{4}} = \frac{\sqrt{x^2-4}}{2}$$ (+1)

start (+.5) used △ or Pyth

So the answer is

$$\frac{1}{2}\left(\frac{x}{2}\right)\frac{\sqrt{x^2-4}}{2} - \frac{1}{2}\ln\left|\frac{x}{2} + \frac{\sqrt{x^2-4}}{2}\right| + C$$

plugged in (+.5)

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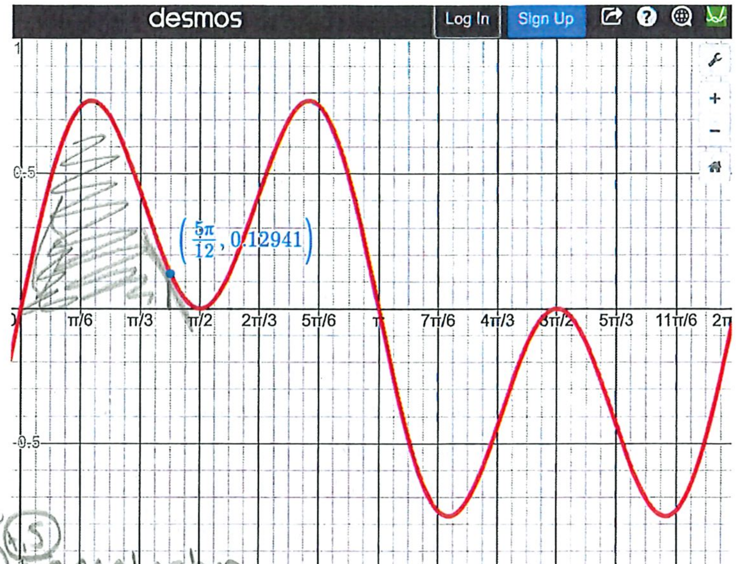
8. A particle moves in a straight line with velocity $v(x) = 2\sin(x)\cos^2(x)$ (graphed below).



(a) [1] (Word Problem #1) What is the velocity when $x = \frac{5\pi}{12}$?

.12941 $\frac{\text{dist unit}}{\text{time unit}}$

(b) [2] (Word Problem #1) Is the acceleration positive or negative when $x = \frac{5\pi}{12}$? Provide justification.

(+.5) [negative - when $x = \frac{5\pi}{12}$

(+1) [The velocity is reducing its' y-value/slope of line tangent is negative $= v'(t) =$ acceleration (+.5)

(c) [3] (Quiz5 #3) Find a formula for the particle's acceleration at time t.

acceleration$(t) = v'(t) = 2\sin(x) \cdot 2\cos(x)(-\sin(x)) + 2\cos(x) \cdot \cos^2(x)$

(+.5) product rule (+1) chain rule (+1)

$= -4\sin^2(x)\cos(x) + 2\cos^3(x)$ got it (+.5)

(d) [2] Is the distance traveled positive of negative when $x = \frac{5\pi}{12}$? Provide justification.

positive (+.5) (+1) [Distance corresponds to the area of the shaded region above - which is above the x-axis $\Rightarrow$ positive

or dist $= \int_0^{5\pi/12} v(t)\,dt$ (+.5)

(e) [2] Is there a time when the particle's net distance traveled is zero? Why or why not? (+.5) yup when area above the x-axis matches

(+.5) [area below the x-axis. looks like @ 2π works (+1)

(f) [4] (WrittenHW7.1 #75) Find a formula for the net distance the particles has traveled in the first t seconds.

Dist $= \int_0^t v(x)\,dx = \int_0^t 2\sin(x)\cos^2(x)\,dx = 2\int_0^t \cos^2(x)\sin(x)\,dx$ (+.5) (+.5)

notation (+.5)
method (+.5)
correctly (+1)

$u = \cos(x)$
$du = -\sin(x)\,dx$
$-du = \sin(x)\,dx$

$2\int \cos^2(x)\sin(x)\,dx = 2\int u^2(-1)\,du = -2\int u^2\,du = -2\,\frac{1}{3}u^3 + C$

$= -\frac{2}{3}\cos^3(x) + C$

So $2\int_0^t \cos^2(x)\sin(x)\,dx = -\frac{2}{3}\cos^3(x)\Big]_0^t = -\frac{2}{3}\cos^3(t) - -\frac{2}{3}\cos^3(0)$

(+.5)

$= -\frac{2}{3}\cos^3(t) + \frac{2}{3}$ got it (+.5)

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