# Word Problem Practice <small>take 2</small>

1. Capacitors are "real life differentiators", specifically $i = C\dfrac{dv}{dt}$,

   - i is the instantaneous current,
   - C is the capacitance (measured in Farads), and
   - $\frac{dv}{dt}$ is the instantaneous rate of voltage change

   If the voltage of a capacitor with capacitance 2μF in a given circuit is $v(t) = 5\cos(100t)$ volts, find the capacitor's instantaneous current response at time t.

2. Test makers use item response functions $P(x)$ to determine the difficulty and effectiveness of a given test question. The variable x is the ability of a test taker and $P(x)$ is the probability that the test taker gets the problem correct. By convention we let an "average ability" correspond with $x = 0$. Thus $P(0) = .75$ means that a person with average ability has a 75% chance of getting the question correct.

   (a) Do you expect $P'(x)$ to be greater than or less than zero? Justify yourself.

   (b) Why might a test maker prefer a question with $P'(0) = 3$ as opposed to one with $P'(0) = .2$?

3. A particle moves according to $s(t) = \cos(t\pi/4)$ (inches from fixed location).

   (a) Find the velocity at time t
   (b) When is the particle at rest?
   (c) When is the particle moving in the positive direction?
   (d) Find the acceleration at time t after 3 seconds.

4. Use linear approximation to estimate a root of $y = x^3 - 4x^2 - 3x + 22$. A section of the graph is provided on the right. Explain your reasoning and steps.
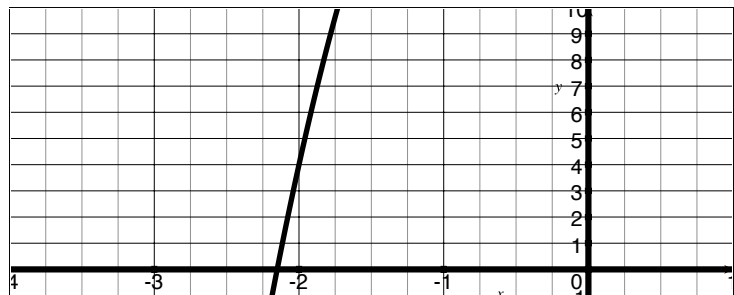
5. A firetruck, parked 15 ft from the base of a building, extended its ladder 40 ft and leaned it against a tall building. After completing their training exercise, the driver gets into the truck and forgets to bring down the ladder before driving away at 5mi/hr (about 7 ft/sec). For the purposes of this problem, assume initially that the base of the ladder is 8 ft above the ground and 15ft from the building. How fast is the top of the ladder moving down the building when the firetruck/base of the ladder is 20 ft from the base of the building? 35 ft?

6. Consider the firetruck described in the previous problem. What rate is the angle between the base of the ladder and the top of the fire truck decreasing when the ladder is 35 ft from the building?

7. Some crystals are easy to grow in the shape of cubes by allowing a solution of water and sodium chlorate to evaporate slowly. Let V be the volume of such a cube with side length x.

   (a) calculate dV/dx when $x = 5$mm and explain its meaning.
   (b) calculate dV/dt when $x = 5$mm and $dx/dt = 2$ mm/sec and explain its meaning.

8. A rectangle's length is increasing at a rate of 8cm/s and its width is increasing at a rate of 3cm/s. When the length is 20cm and the width is 10cm, how fast is the areas of the rectangle increasing? Find a formula for how fast the area of the rectangle increases when the width of the rectangle is w and the length is l.

9. If C is the cost ($ out) a company incurs by producing x units of their commodity, the marginal cost MC is equal to $\lim\limits_{\Delta x \to 0} \dfrac{\Delta C}{\Delta x} = \dfrac{dC}{dx}$. Similarly, if R is the revenue ($ in) a company gathers by producing x units on their commodity, the marginal revenue MR is equal to $\frac{dR}{dx}$. Also if P is profit ($), the marginal profit, MP, is $\frac{dP}{dx}$.

   Recall that Profit=($ in)-($ out)=Revenue-Cost. If a companies cost function $C(x) = 10,000 + 3x^2$ and revenue function $R(x) = 450x$, find the number x units that should be produced to maximize profit. Explain why economics care when $MP = 0$.

10. If a current I passes through a resistor with resistance r, Ohm's Law implies that the voltage (V) drop is RI. Assume that voltage remains a constant 20 volts. An unreliable resistor claims a resistance of 10 ohms but may be off by up to 1.5 ohms. Use the linear approximation to approximate the error when calculating i.

11. Use the graph provided by JCRooks on CoronavirusWA Reddit on Jan 24th duplicated below. JCRooks is plotting the Slope of the 7-day average of new Covid-19 cases in King County over time.
    (a) [2] Describe what is happening to the 7-day average of new Covid-19 cases in King County in Feb. 2021.

    (b) [3] Identify a time that the 7-day average of new Covid-19 cases peaked. Explain/justify your answer.

Slope of 7-Day Avg