Word Problem Practice

1. Test makers use item response functions $P(x)$ to determine the difficulty and effectiveness of a given test question. The variable x is the ability of a test taker and $P(x)$ is the probability that the test taker gets the problem correct. By convention we let an "average ability" correspond with $x = 0$. Thus $P(0) = .75$ means that a person with average ability has a 75% chance of getting the question correct.

   (a) Find $\lim_{x\to\infty} P(x)$ and explain it's meaning.

   (b) Assume the question is a True/False question, find $\lim_{x\to-\infty} P(x)$. Justify yourself.

   (c) Assume the question is multiple choice with 4 choices, find $\lim_{x\to-\infty} P(x)$.

2. In algorithms class you are given an assignment to sort n integers within a given range. You would like to use bubble sort or bucket sort. The bubble sort algorithm's efficiency is dependent on n and in a worse case scenario could take n^2 steps. Computer scientists would say, that bubble sort is "big O" of n^2. By contrast, bucket sort is big O of n. Generally to determine if one algorithm is better than another, computer scientists consider the limit as n goes to infinity of the ratio of the two big O's. Perform this operation and compute $\lim_{n\to\infty} \frac{n}{n^2}$ and use the answer to determine which algorithm is better.

3. In the theory of relativity, the Lorentz contraction formula

$$L = L_0\sqrt{1 - \frac{v^2}{c^2}}$$

   expresses the length L of an object as a function of its velocity v with respect to an observer, where L_0 is the length of the object at rest and c is the speed of light. Find $\lim_{v\to c^-} L$ and interpret the result as a physicist.

4. Explain how scientists know there are at least two points directly opposite each other on the surface of the earth that are the same temperature.

5. A tank contains 8000L of pure water. Brine that contains 30 g of salt per liter of water is pumped into the tank at a rate of 25L/min. Find a function that records the concentration of salt after t minutes (in grams per liter) and then find out what happens to the concentration as $t \to \infty$.

6. Consider a model of motion first advanced by Galileo. Galileo's Law states that the distance a freely falling object falls is proportional to the square of the time it has fallen. Let us take $s(t)$ to be the distance an object falls in t seconds from the top of a building 100 meters in height. The proportionality constant in Galileo's Law can be determined, by experiment, to be 4.9 meters per second per second near the Earth's surface. We thus have that $s(t) = 4.9t^2$. Find the velocity of the object after 3 seconds. When does the object hit the ground?

7. If a rock is thrown upward on the planet Mars with a velocity of 10m/s, its height is meters t seconds later is given by $h(t) = 10t - 1.86t^2$. Find the instantaneous velocity of the rock after one second. Find when the rock hits the max height.

8. Recall Newton's Law of Cooling: If D_0 is the initial temperature difference between an object and its surroundings, and if its surroundings have temperature T_s, then the temperature of the objects at time t is modeled by the function

$$T(t) = T_s + D_0 e^{-kt}$$

where k is a positive constant that depends on the type of object.
Find $\lim_{t\to\infty} T(t)$ and interpret the result as a scientist.

9. A logistic growth model is a function that often describes the size of a population at time t and is of the form

$$f(t) = \frac{c}{1 + ae^{-bt}}$$

where a, b, and c are positive constants specific to the population under study.
Find $\lim_{t\to\infty} f(t)$ and interpret the result as a biologists.

10. The figure below shows a fixed circle C_1 with equation $(x-1)^2 + y^2 = 1$ and a shrinking circle C_2 with radius r and center the origin. P is the point $(0, r)$, Q is the upper point of intersection of the two circles, and R is the point of intersection of the line PQ and the x-axis. What happens to R as C_2 shrinks, that is, as $r \to 0^+$?