

Note: This is a practice final and is intended only for study purposes. The actual exam will contain different questions and may have a different layout.

1. TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let f and g be differentiable functions and h be a constant.

T F $\frac{x+h}{2x} = \frac{1+h}{x}$

T F $\sqrt{x^2 + h^2} = x + h$

T F $\lim_{x \rightarrow r} f(x) = f(r)$ for all r in the domain of f .

T F If $\lim_{x \rightarrow r} g(x) = 0$, then $\lim_{x \rightarrow r} \frac{f(x)}{g(x)}$ does not exist.

T F $\frac{d}{dx} \left(\frac{1}{x} \right) = -1$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

2. Sketch the graph and then *find the formula* of an example function f that satisfies the following conditions:

(a) $f(2) = 2$

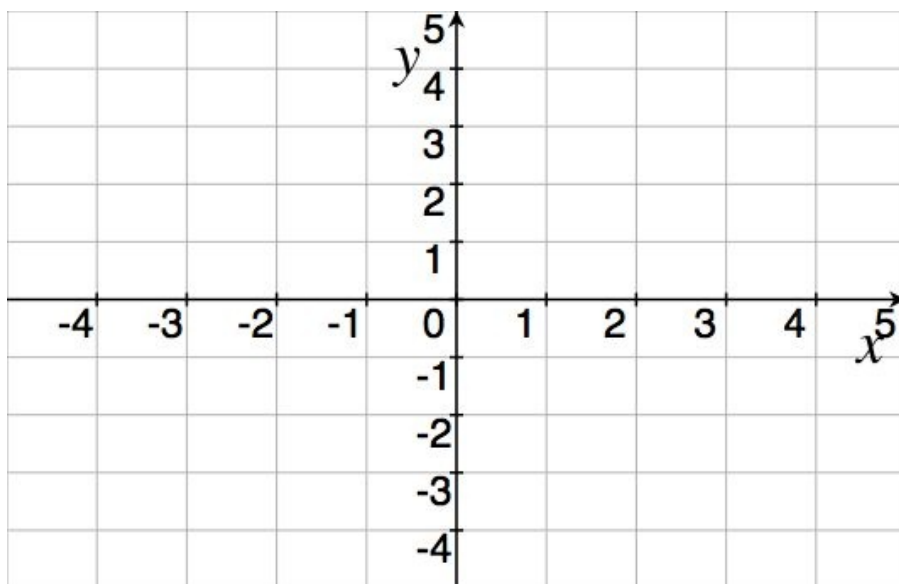
(b) $\lim_{x \rightarrow 2} f(x) = -4$

(c) f is not differentiable when $x = -3$

(d) f is continuous when $x = -3$

(e) $\lim_{x \rightarrow 0^+} f(x) = \infty$

(f) $f'(4) = 2$



3. Compute the following limits:

$$(a) \lim_{x \rightarrow 1} \frac{x^2 + x - 2}{2x^2 - 8x + 6}$$

$$(b) \lim_{x \rightarrow \infty} \frac{x^2 + x - 2}{2x^2 - 8x + 6}$$

$$(c) \lim_{\theta \rightarrow 0^+} \frac{\theta + \theta^2}{1 - \cos \theta}$$

$$(d) \lim_{x \rightarrow \infty} x \sin \left(\frac{5\pi}{x} \right)$$

$$(e) \lim_{x \rightarrow 0} x^4 \sin \left(\frac{1}{x} \right)$$

$$(f) \lim_{x \rightarrow 1} \frac{1}{x - 1}$$

4. Let $f(x) = \begin{cases} \sqrt{1 - (x + 3)^2} & \text{if } -4 \leq x \leq -2 \\ 1 & \text{if } -2 < x < 1 \\ -(x - 2)^2 + 2 & \text{if } 1 < x \end{cases}$

Graph $f(x)$ and then sketch the graph $f'(x)$ below on its own set of axes. Afterwards, answer the following questions.

(a) $\lim_{x \rightarrow 1} f(x)$

(b) $\lim_{x \rightarrow 3} [4f(x) - 7]$

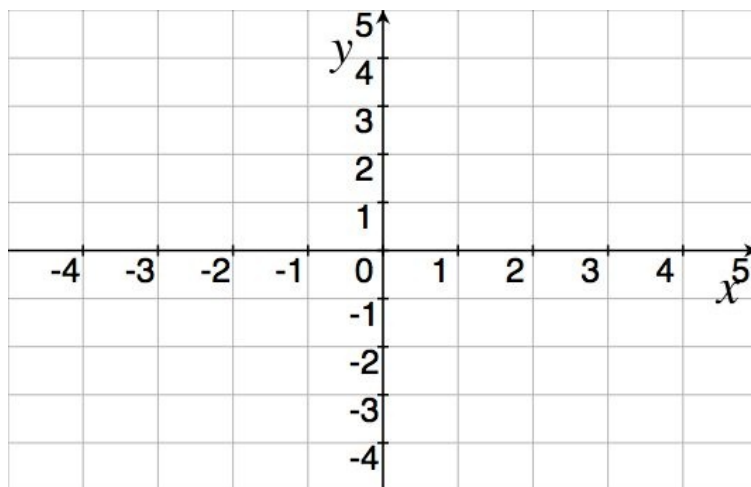
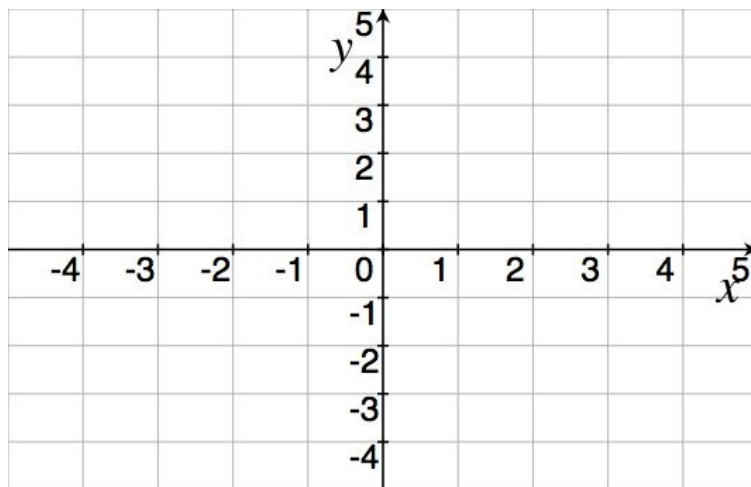
(c) $\lim_{x \rightarrow -2} f(x)$

(d) $\lim_{x \rightarrow -2^-} f(x)$

(e) $\lim_{x \rightarrow 3} f'(x)$

(f) $\lim_{x \rightarrow \infty} f(x)$

(g) $[f + f]'(2)$



5. Compute the derivatives of the following functions. You do *not* need to simplify.

(a) $f(x) = x^3 + 3^x + \pi^\pi$

(b) $g(t) = \ln(t) \left(\frac{2 + t^2}{3t - 1} \right)$

(c) $h(\theta) = 7 \sec(\sqrt{\theta})$

(d) $y = \sqrt{x}e^{x^7}(x^6 + 3)^{10}$

(c) $y = (\cos(x))^x$

(d) $x^2y^2 = 4 - y \arctan(5x)$

6. Find the equation of the line tangent to the graph of f when $x = 2$ if $f(x) = m(n(x))$, $n(2) = -1$, $m(-1) = 6$, $n'(2) = 3$, and $m'(-1) = 5$.

7. Find the antiderivative for each of the following functions:

(a) $2x - x^3 + 7 \sin(x)$

(b) $\frac{5 - 4x^3 + 2x^6}{x^6}$

8. Consider the function $f(x) = \sqrt[3]{x}$

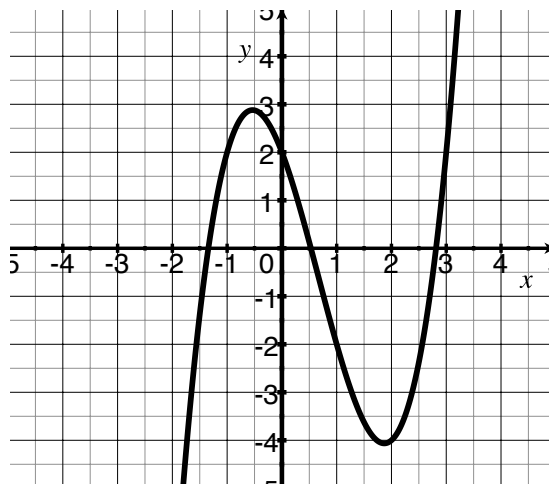
(a) Evaluate the integral $\int_1^8 \sqrt[3]{x} dx$

- (b) Draw a picture that corresponds to the area you computed in (a).

9. Find the linearization of $f(x) = \frac{1}{\sqrt{x}}$ that is parallel to the line $y - 3 = \frac{-27}{2}(x + 5)$

10. Consider the graph of $f(x) = x^3 - 2x^2 - 3x + 2$ graphed to the right..

- (a) [3] (favorite problem!) Find the equation of the line tangent to the graph of f when $x = -1$.

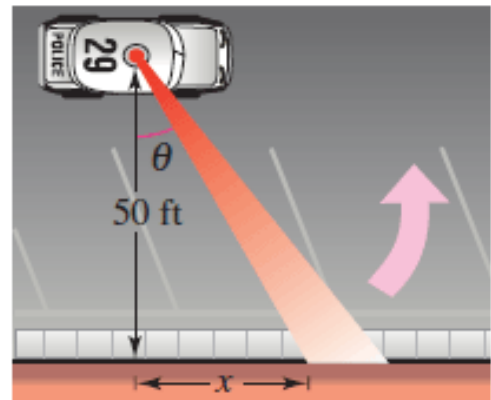


- (b) [2] Use linear approximation to estimate the negative root of f .

11. A photographer is taking a picture of a 4-foot painting hung in an art gallery. The camera lens (positioned on a tripod) is 1 foot below the lower edge of the painting. How far should the camera be from the painting to maximize the angle subtended by the camera lens?

12. (WebHW11 #9) A patrol car is parked 50 feet from a building shown to the right. The revolving light on top of the car turns at a rate of 8 revolutions per minute.

(a) [1] Find θ as a function of x .



(b) [3] Find how fast the light beam is moving along the wall when the beam makes an angle of 30° with the building wall.

13. Find the dimensions of the rectangle of largest area that has its base on the x -axis and its other two vertices above the x -axis and lying on the parabola $y = 7 - x^2$

14. A truck has a minimum speed of 9 mph in high gear. When traveling x mph, the truck burns diesel fuel at the rate of

$$0.003935 \left(\frac{675}{x} + x \right) \frac{\text{gal}}{\text{mile}}$$

Assume that the truck can not be driven over 63 mph, that diesel fuel costs \$2.84 a gallon, and that the driver is paid \$12 an hour. Find the speed that will minimize the cost of a 500 mile trip.