

Note: This is a practice midterm and is intended only for study purposes. The actual exam will contain different questions and perhaps a different layout.

1.  TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let  $f$  and  $g$  be functions.

T F  $\frac{d}{dx}b^c = cb^{c-1}$  for a fixed  $b$  and  $c$

T F  $(x + y)^2 = x^2 + y^2$

T F  $\frac{d}{dx}2^x = x2^{x-1}$

Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

2.  Sketch the graph of an example function  $f$  that satisfies the following conditions:

(a)  $f$  is not differentiable when  $x = -3$

(b)  $f$  is continuous when  $x = -3$

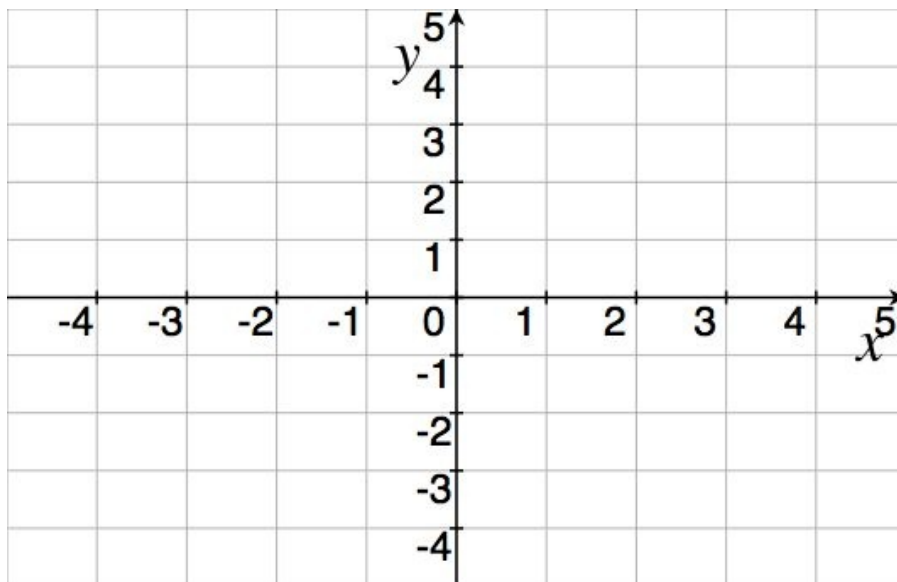
(c)  $f(1) = -4$

(d)  $\lim_{x \rightarrow 1} f(x) = 2$

(e)  $f'(3) = -\frac{1}{2}$

(f)  $\lim_{x \rightarrow -1^+} f(x) = 0$

(g)  $\lim_{x \rightarrow -1^-} f(x) = 1$



3. Find a formula for the function  $f$  you drew in problem (2).

4. Find the following:

$$\lim_{x \rightarrow 0} \frac{3 \sin(4x)}{2 \sin(3x)}$$

$$\lim_{x \rightarrow 0} \frac{\cos x - 1}{\sin x}$$

5. Suppose that  $f(2) = -3$ ,  $g(2) = 4$ ,  $f'(2) = -2$ , and  $g'(2) = 7$ . Find  $h'(2)$  where  $h$  is:

$$h(x) = 5f(x) - 4g(x)$$

$$h(x) = f(x)g(x)$$

$$h(x) = \frac{f(x)}{g(x)}$$

$$h(x) = \frac{g(x)}{1+f(x)}$$

6. If  $F(x) = f(g(x))$ , where  $f(-2) = 8$ ,  $f'(-2) = 4$ ,  $f'(5) = 3$ ,  $g(5) = -2$ , and  $g'(5) = 6$ , find  $F'(5)$ .

7. Find the  $\frac{dy}{dx}$  of the following and do *not* simplify:

$$y = (2x^2 + 7x^2)(3^x - 2^x)$$

$$y = \frac{\sin(x) + x^2 \cos(x)}{\cos(x)}$$

$$y = \sqrt{\frac{x^2 + 1}{4x^5 - 3x}}$$

$$e^y \sin(x) = x + xy$$

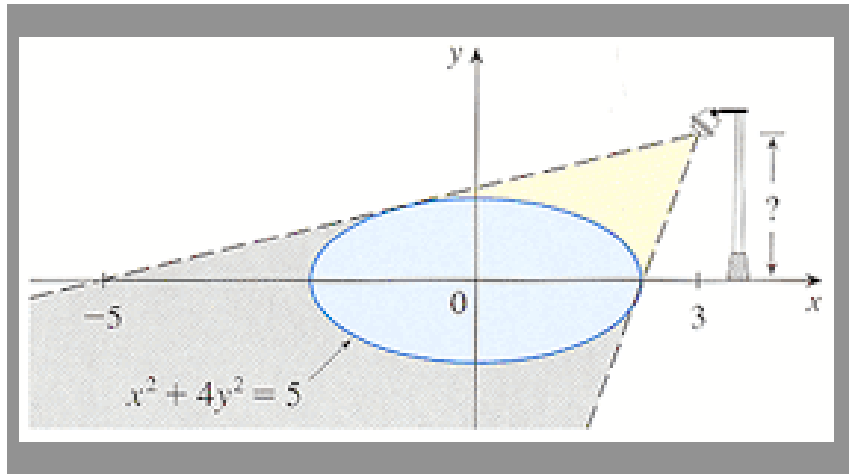
$$y = \sin(e^{\sin(x^2)})$$

$$y = (\sin x)^{\ln e^2}$$

8. Find the equations of all lines tangent to the curve described by the relation  $x^2y^2 + xy = 2$  that are also parallel to the line described by  $y = -x - \pi$ .

9. [10] (§3.5 #69) The figure below shows a lamp located three units to the right of the  $y$ -axis and a shadow created by the elliptical region  $x^2 + 4y^2 \leq 5$ . The point  $(-5, 0)$  is on the edge of the shadow.

- (a) [2] Find  $\frac{dy}{dx}$  of the ellipse.



- (b) [3] Denote the point that is both on the ellipse and the top dashed line by  $(c, d)$ . Notice that the slope of the top dashed line is thus  $\frac{d - 0}{c - (-5)}$ . Use this information and what you found in part (a) to find the value of  $c$ .

- (c) [5] Find the equation of the top dashed line and then find out the height of the lamp.

10. [] (Story Problem Worksheet) Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit. No, doing both questions will not earn you extra credit.

(a) (§3.9 #21) [5] Ryan and Stella were being chased by a pack of zombies. At point  $P$  they decided to split up and Stella ran south at 12 ft/s. Ryan waited for ten seconds to try to draw most of the zombies towards him and then started to run east at 15 ft/s. One minute later the two of them are still alive and running in their respective directions. At what rate are Ryan and Stella moving apart at this instant?

(b) If a current  $i$  passes through a resistor with resistance  $r$ , Ohm's Law states that the voltage drop is  $v = ri$ . Assume that voltage remains a constant 20 volts. An unreliable resistor claims a resistance of 10 ohms but may be off by up to 1.5 ohms. Use *linear approximation* to estimate the error when calculating  $i$ .