EXAM 2 — TMath 124 — Practice

Show *all* your work (numerically, algebraically, or geometrically) for the following problems. Supporting work is needed to earn credit.

1. Find both the following:

$$\lim_{x\to 0}\frac{\sin(4x)}{6x} \qquad\qquad \left(\frac{\sin(4x)}{6x}\right)'$$

algebraically:

$$\lim_{x\to 0}\frac{\sin(4x)}{(6x)}\cdot\frac{4}{4} = \lim_{x\to 0}\frac{\sin 4x}{4x}\cdot\frac{4}{6}$$

$$= \underbrace{\lim_{x\to 0}\frac{\sin 4x}{4x}}_{1}\cdot\lim_{x\to 0} 4/6 = 2/3$$

graphically:  $(0, 2/3)$ $\Rightarrow 2/3$

derivative? Quotient rule

$$\frac{6x\cdot(\sin(4x))' - \sin(4x)(6x)'}{(6x)^2}$$

chain rule

$$\frac{4\cdot 6x\cos(4x) - 6\sin(4x)}{36x^2}$$

2. Sketch the graph of an example function f that satisfies the following conditions:

(a) f is not differentiable when $x=-3$

(b) f is continuous when $x=-3$

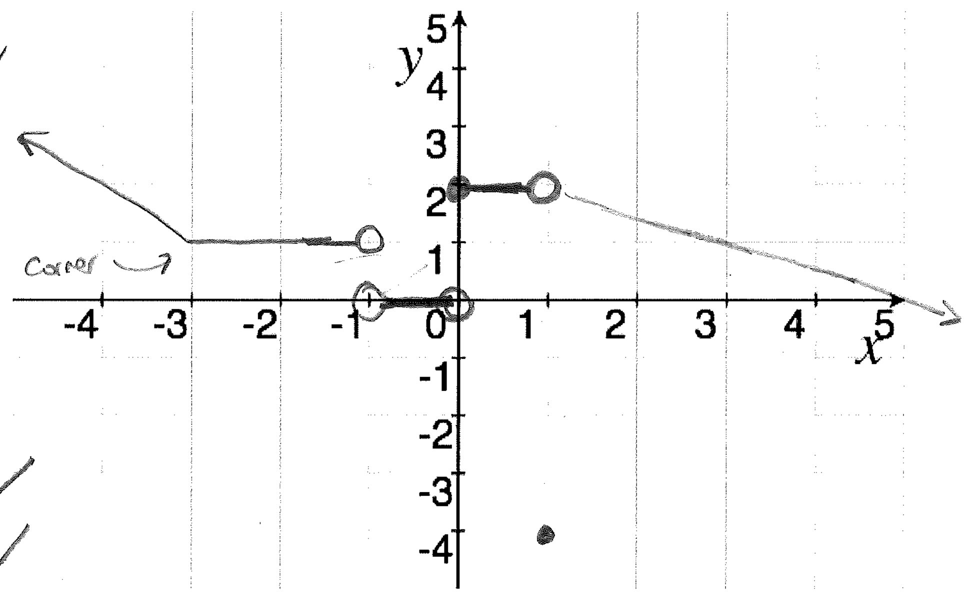
(c) $f(1)=-4$

(d) $\lim_{x\to 1} f(x) = 2$

(e) $f'(3) = -\frac{1}{2}$

(f) $\lim_{x\to -1^+} f(x) = 0$

(g) $\lim_{x\to -1^-} f(x) = 1$



3. Find a formula for the function f you drew in problem (2).

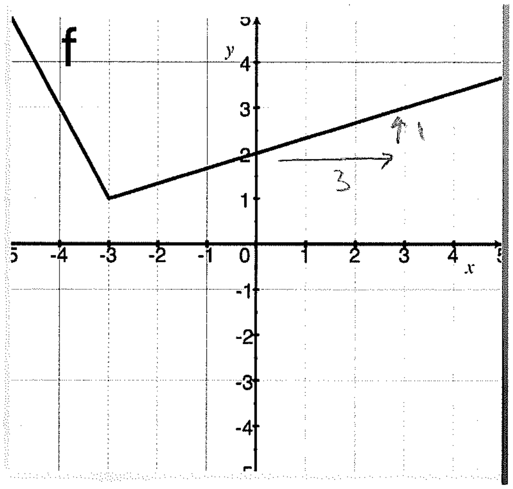
☺ lots of straight lines

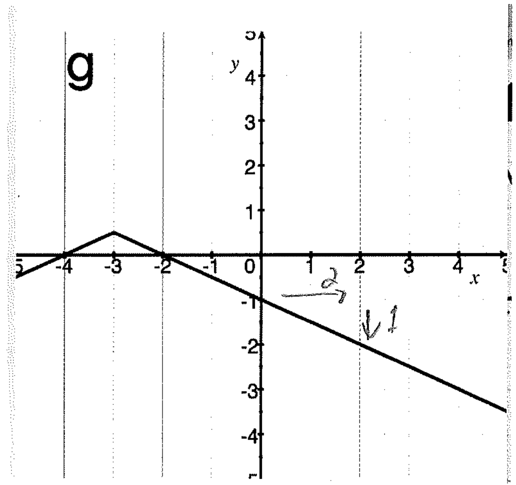
$$f(x) = \begin{cases} -x-2 & \text{if } x \le -3 \\ 1 & \text{if } -3 < x < -1 \\ 0 & \text{if } -1 < x < 0 \\ 2 & \text{if } 0 \le x < 1 \\ -4 & \text{if } x = 1 \\ -\frac{1}{2}x + \frac{5}{2} & \text{if } 1 < x \end{cases}$$

4. Identify which derivative rule(s) you can use to find $\frac{dy}{dx}$. Do *not* find $\frac{dy}{dx}$!!

	Derivative Rule(s)
$y = \sqrt{\frac{x-1}{x^4+1}}$	logarithmic differentiation ln both sides ln properties implicitly diff $\ln(y) = \ln\left(\frac{x-1}{x^4+1}\right)^{1/2}$ $\ln y = \frac{1}{2}[\ln(x-1) - \ln(x^4+1)]$ $\ln y = \frac{1}{2}\ln(x-1) - \frac{1}{2}\ln(x^4+1)$ OR Chain rule outside $u^{1/2}$ inside ↳ need quotient rule
$y + x4^y = x^9$	differentiate each term use product rule on x and 4^y use implicit differentiation with power rule OR algebra to $x4^y = x^9 - y$ use logarithmic diff $\ln(x4^y) = \ln(x^9 - y)$ $\ln(x) + y\ln(4) = \ln(x^9 - y)$
$y = e^{x^3-5x}$	Chain Rule outside function e^u inside function $x^3 - 5x$ OR logarithmic diff $\ln(y) = \ln(e^{x^3-5x})$ simplify $\ln(y) = x^3 - 5x$ then implicitly diff
$y = (\tan(x))^x$	logarithmic differentiation $\ln(y) = \ln((\tan(x))^x)$ use ln prop. to simplify $\ln(y) = x\ln(\tan(x))$ use implicit diff + the product rule

5. Use the graphs of f and g below for the following questions.





(a) Find an x so that $g'(x)$ does not exist.

at corners so $x = -3$

(b) Estimate $\frac{d}{dx}(f(x)g(x))|_{x=0}$

$$f(0)\,\frac{d}{dx}(g(x))\Big|_{x=0} + \frac{d}{dx}(f(x))\Big|_{x=0}\cdot g(0)$$

$$= 2\,\frac{-1}{2} + \frac{1}{3}\cdot(-1) = -1 - \frac{1}{3} = -\frac{4}{3}$$

(c) If $c(x) = f(g(x))$, then estimate $c'(4)$.

$$c'(4) = f'(g(4))\cdot g'(4)$$

$$= \underline{f'(-3)}\,(-\tfrac{1}{2})$$

$\hookrightarrow$ DNE ?

(d) If $h(x) = g(3x-1)$, then estimate $h'(2)$.

$$h'(2) = g'(3(2)-1)\cdot[3x-1]'\Big|_{x=2}$$

$$= g'(5)\cdot 3$$

$$= -\tfrac{1}{2}\cdot 3 = -\tfrac{3}{2}$$

6. The differentiable functions f and g are defined for all real numbers. Values for f, f', g, and g' for various x values are given in the table.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	3	4	2	6
2	1	5	8	7
3	7	7	2	9

(a) Given that $h(x) = \frac{f(x)}{2x+g(x)}$, find $h'(1)$.

$h'(1) = \frac{[2(1)+g(1)]f'(1) - f(1)[2+g'(1)]}{[2(1)+g(1)]^2}$

$= \frac{(2+2)\cdot 4 - 3(2+6)}{(2+2)^2} = \frac{16-3\cdot 8}{4^2} = \frac{16-24}{16} = \frac{-8}{16} = -\frac{1}{2}$

(b) Find the linearization of f at $x = 2$.

Looking for $y - y_1 = m(x - x_1)$

m = slope of line tangent to f @ $x = 2$

$= f'(2) = 5$

So $y - 1 = 5(x-2)$

OR

$y = 5x - 9$

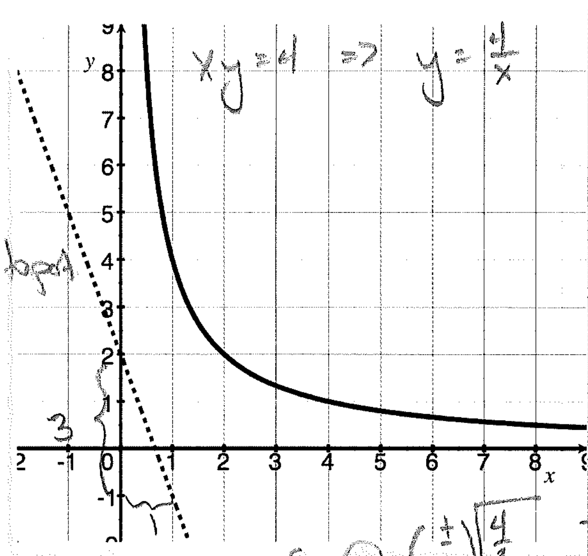
(c) Use the linearization of f to approximate $f(2.05)$.

plug in 2.05 to the line eq above

$5(2.05) - 9 = 1.25$

7. A particle moves along a hyperbola $xy = 4$ when $x > 0$. The graph is shown below with a solid curve. The dotted line is of a dust particle moving along a straight line.

(a) Find the point that the particle's movement is parallel to a dust particle moving along the dotted straight line graphed.



ie find x when

slope of dotted line = slope of line tang to graph

$\frac{-3}{1} = y'$

$y = \frac{4}{x} = 4x^{-1} \Rightarrow y' = -4x^{-2}$

So $-3 = \frac{-4}{x^2}$

$\Rightarrow -3x^2 = -4$

$x^2 = 4/3$

$x = \pm\sqrt{4/3}$

So @ $\left(\pm\sqrt{\frac{4}{3}}, \frac{4}{\pm\sqrt{4/3}}\right)$

(b) When the particle reaches an x value of 1, the y-coordinate is decreasing at a rate of 3 cm/s. How fast is the x-coordinate of the point changing at that instant?

$\hookrightarrow \frac{dy}{dt} = -3$ cm/s

WANT $\left.\frac{dx}{dt}\right|_{x=1}$

$x \cdot y = 4$

$\frac{d}{dx}$

$x\frac{dy}{dt} + \frac{dx}{dt}y = 0$

when $x = 1$

$\left(y = \frac{4}{x} \text{ so } y = 4\right)$

$1\cdot(-3) + \left.\frac{dx}{dt}\right|_{x=1} \cdot y|_{x=1} = 0$

$-3 + \left.\frac{dx}{dt}\right|_{x=1} \cdot 4 = 0$

$\Rightarrow \left.\frac{dx}{dt}\right|_{x=1} = \frac{3}{4}$

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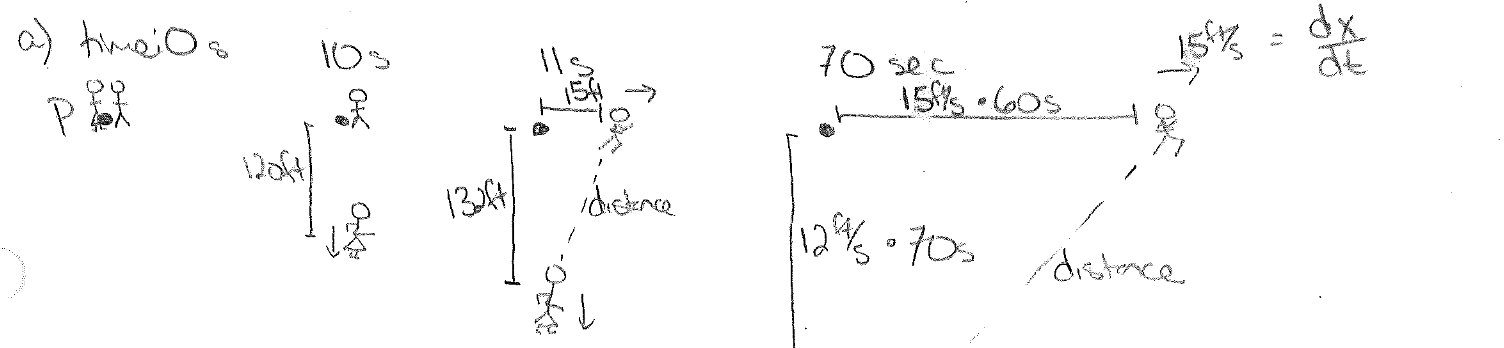
9. [5] Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit.
No, doing both questions will not earn you extra credit.

(a) (§3.9 #21) [5] Ryan and Stella were being chased by a pack of zombies. At point P they decided to split up and Stella ran south at 12 ft/s. Ryan waited for ten seconds to try to draw most of the zombies towards him and then started to run east at 15 ft/s. One minute later the two of them are still alive and running in their respective directions. At what rate are Ryan and Stella moving apart at this instant?

(b) ~~A man walks along a straight path at a speed of 4ft/s. A searchlight is located on the ground 20 ft from the path and is kept focused on the man. At what rate is the searchlight rotating when the man is 15ft from the point on the path closest to the searchlight?~~

a)



$$\begin{pmatrix}\text{distance}\\ \text{between}\\ \text{them}\end{pmatrix}^2 = \begin{pmatrix}\text{distance}\\ \text{Ryan runs}\end{pmatrix}^2 + \begin{pmatrix}\text{distance}\\ \text{Stella runs}\end{pmatrix}^2 \qquad \downarrow 12\ \text{ft/s} = dy/dt$$

$d^2 = x^2 + y^2$ where x = distance Ryan runs at time t
y = distance Stella runs at time t
d = distance between ryan + stella

we want $\left.\frac{dd}{dt}\right|_{t=70}$

So $\frac{d}{dt}[d^2] = \frac{d}{dt}[x^2+y^2]$

$\Rightarrow \frac{\cancel{2d}\,\frac{dd}{dt}}{\cancel{2d}} = \frac{2x\frac{dx}{dt} + 2y\frac{dy}{dt}}{2d}$

$\Rightarrow \frac{dd}{dt} = \frac{x\frac{dx}{dt} + y\frac{dy}{dt}}{d}$

when $t = 70$
$x = 15\,\text{ft}/\cancel{s} \cdot 60\cancel{s} = 900\text{ft}$
$y = 12\,\text{ft}/\cancel{s} \cdot 70\cancel{s} = 840\text{ft}$
Recall: $d^2 = x^2 + y^2$
$\Rightarrow d = \sqrt{900^2 + 840^2}$

So $\left.\frac{dd}{dt}\right|_{t=70} = \frac{900(15) + 840(12)}{\sqrt{900^2+840^2}} \approx 19.2\ \text{ft/s}$