

Note: This is a practice exam and is intended only for study purposes. The actual exam will contain different questions and may have a different layout.

1. TRUE/FALSE: Let f and g be functions. Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F.

T F $\frac{3x+y}{3z} = \frac{x+y}{z}$

T F $(x+y)^2 = x^2 + y^2$

T F If $\lim_{x \rightarrow a} g(x) = 0$ then $\frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}$ does not exist.

T F If $\lim_{x \rightarrow \infty} f(x) = \infty$ and $\lim_{x \rightarrow \infty} g(x) = \infty$, then $\lim_{x \rightarrow \infty} [f(x) - g(x)] = 0$.

T F If f is continuous at a , then f is differentiable at a .

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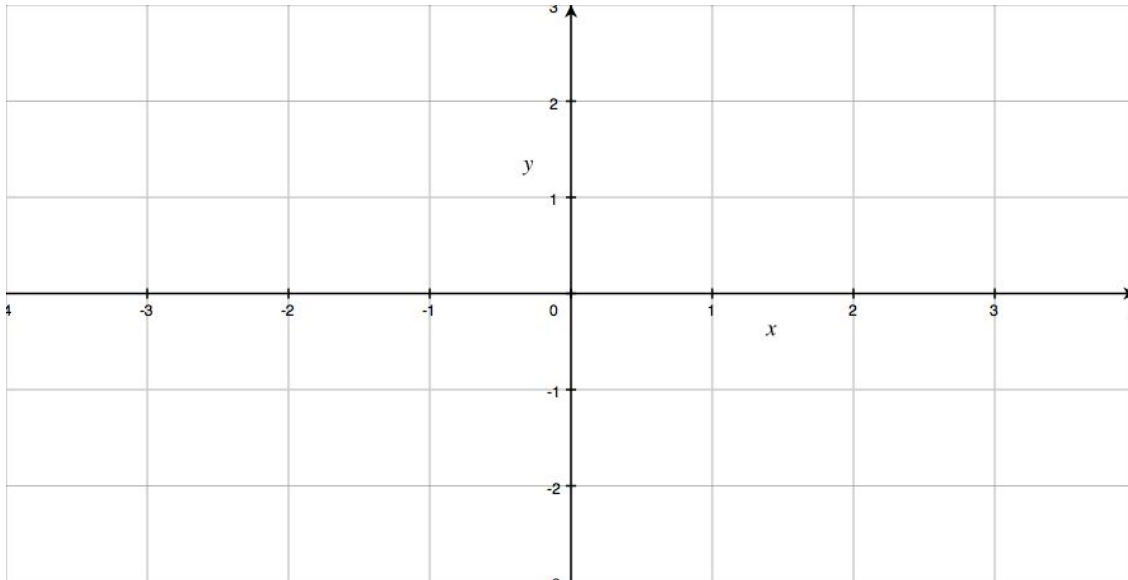
Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

2. Draw a graph and then find a formula for a function $f(x)$, that satisfies all of the following criteria:
- (a) has a vertical asymptote at $x = 2$
 - (b) is not defined at $x = 0$,
 - (c) $\lim_{x \rightarrow -\infty} f(x) = -1$,
 - (d) $f(3) = 2$
 - (e) $f'(3) = 0$

3. Given the rules of f and g below, graph both functions on the axis provided and evaluate the following

$$f(x) = \begin{cases} \frac{1}{x} & \text{if } x < 0, \\ 2 & \text{if } x = 0, \\ \ln x & \text{if } x > 0, \end{cases}$$

$$g(x) = \sin\left(\frac{\pi}{4}x\right)$$



$$\lim_{x \rightarrow \infty} g(x)$$

$$\lim_{x \rightarrow 0} f(x)$$

$$f(0)$$

$$\lim_{x \rightarrow 1} [\pi f(x) \times g(x)]$$

$$\lim_{x \rightarrow -2} (2f(x) + g(x))$$

$$g'(2)$$

List any values that f is *not* continuous at:

Sketch $g'(x)$

4. [] Find the limit if it exists, or explain why it does not exist.

$$\lim_{x \rightarrow 3} \frac{x^2 - 4x + 3}{x^2 - 2x - 3}$$

$$\lim_{x \rightarrow \infty} e^{-2x} \sin x$$

$$\lim_{h \rightarrow 0} \frac{(1+h)^{-1} - 1}{h}$$

$$\lim_{x \rightarrow \infty} \arctan(x^2 - x^4)$$

$$\lim_{x \rightarrow 0} \frac{3x^2}{3 - \sqrt{9 - x^2}}$$

$$\lim_{x \rightarrow 3^+} \ln(x - 3)$$

5. Does $f(x) = 2x^3 + 6x^2 - 10x - 30$ have a root between 2 and 3? Explain your reasoning and cite theorems if you use any.

6. Find the equation for the line tangent to the graph of $y = \frac{1}{(x-2)^2}$, when $x = 3$.

7. Suppose that the motion of a ball can be described by the equation $f(t) = t^2 + t - 3$. Find the instantaneous velocity of the ball after 4 seconds.

8. □ Using the **definition**, find the derivative of $f(x) = \sqrt{2x - \frac{1}{2}}$

9. Describe 3 situations in which a function $f(x)$ could **fail** to be differentiable at a point.

10. [5] (Story Problem Worksheet) Choose *ONE* of the following. Clearly identify which of the two you are answering and what work you want to be considered for credit. No, doing both questions will not earn you extra credit.

- (a) The toll T charged for driving on the 520 bridge depend on the time of day. The Pay by Mail rates are given in the table below. Sketch a graph of T as a function of time t , measured in hours since past midnight. Then point out any discontinuities of this function and describe their significance to someone who uses the road.

| Monday-Fridays | Pay By Mail | Mondays-Fridays | Pay By Mail |
|-----------------|-------------|------------------|-------------|
| Midnight to 5am | 0 | 2pm to 3pm | \$4.30 |
| 5am to 6am | \$3.10 | 3pm to 6pm | \$5.00 |
| 6am to 7am | \$4.30 | 6pm to 7pm | \$4.30 |
| 7am to 9am | \$5.00 | 7pm to 9pm | \$3.75 |
| 9am to 10am | \$4.30 | 9pm to 11pm | \$3.10 |
| 10am to 2pm | \$3.75 | 11pm to Midnight | 0 |

- (b) The total cost of repaying a student loan at an interest rate of $r\%$ per year is $C = f(r)$.
- What is the meaning of the derivative $f'(r)$?. What are its units?
 - What does the statement $f'(10) = 1200$ mean?
 - Is $f'(r)$ always positive or does it change sign?