

Note: This is a practice exam and is intended only for study purposes. The actual exam will contain different questions and may have a different layout.

1.  TRUE/FALSE: Let  $f$  and  $g$  be functions. Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F.

T F  $\frac{3x+y}{3z} = \frac{x+y}{z}$

T F  $(x+y)^2 = x^2 + y^2$

T F  $\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}$  for all  $a$

T F If  $\lim_{x \rightarrow \infty} f(x) = \infty$  and  $\lim_{x \rightarrow \infty} g(x) = \infty$ , then  $\lim_{x \rightarrow \infty} [f(x) - g(x)] = 0$ .

T F If  $f$  is continuous at  $a$ , then  $f$  is differentiable at  $a$ .

T F If  $f$  is differentiable at  $a$ , then  $f$  is continuous at  $a$ .

Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

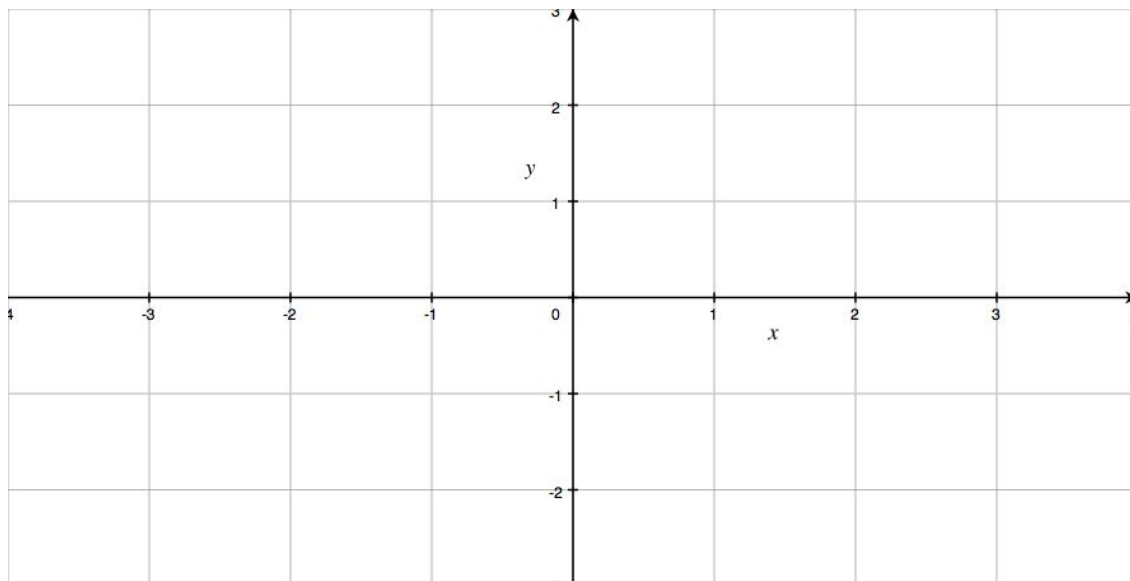
2. Find a formula for a function that satisfies all of the following criteria:

- has a vertical asymptote at  $x = 2$
- a removable discontinuity at  $x = 0$ , and
- horizontal asymptote at  $y = -1$ .

3. Given the rules of  $f$  and  $g$  below, graph both functions on the axis provided and evaluate the following

$$f(x) = \begin{cases} \frac{1}{x} & \text{if } x < 0, \\ 2 & \text{if } x = 0, \\ \ln x & \text{if } x > 0, \end{cases}$$

$$g(x) = \sin\left(\frac{\pi}{4}x\right)$$



$$\lim_{x \rightarrow \infty} g(x)$$

$$\lim_{x \rightarrow 0} f(x)$$

$$f(0)$$

$$\lim_{x \rightarrow 1} [\pi f(x) \times g(x)]$$

$$\lim_{x \rightarrow -2} (2f(x) + g(x))$$

$$g'(2)$$

List any values that  $f$  is *not* continuous at:

Graph  $g'(x)$

4. [] Find the limit if it exists, or explain why it does not exist.

$$\lim_{x \rightarrow 3} \frac{x^2 - 4x + 3}{x^2 - 2x - 3}$$

$$\lim_{x \rightarrow \infty} e^{-2x} \sin x$$

$$\lim_{h \rightarrow 0} \frac{(1+h)^{-1} - 1}{h}$$

$$\lim_{x \rightarrow \infty} \arctan(x^2 - x^4)$$

$$\lim_{x \rightarrow 1} \frac{2x - 2}{|x - 1|}$$

$$\lim_{x \rightarrow 3^+} \ln(x - 3)$$

5. Does  $f(x) = 2x^3 + 6x^2 - 10x - 30$  have a root between 2 and 3? Explain your reasoning and cite theorems if you use any.

6.  Find the equation for the line tangent to the graph of  $y = \frac{1}{(x-2)^2}$ , when  $x = 3$ .

7. Suppose that the motion of a ball can be described by the equation  $f(t) = t^2 + t - 3$ . Find the instantaneous velocity of the ball after 4 seconds.

8. □ Using the **definition**, find the derivative of  $f(x) = \sqrt{2x - \frac{1}{2}}$

9. Describe 3 situations in which a function  $f(x)$  could **fail** to be differentiable at a point.