Mean Value Theorem §4.2

1. Consider the function $f(x) = \cos 2x$ with a domain of $[\pi/8, 15\pi/8]$.

   (a) Verify the three hypotheses of Rolle's Theorem.

   (b) Find all numbers c that satisfy the conclusion of Rolle's Theorem.

2. Exhibit the Mean Value Theorem for $y = x^3 + x - 1$ on the interval $[1, 2]$.

Notice 1) $y = x^3 + x - 1$ is a cubic polynomial which are known to be cont.

2) $y' = 3x^2 + 1$ so $y = x^3 + x - 1$ is differentiable.

The mean value theorem implies there exists a c between 1 and 2 so that $f'(c) = \frac{f(2) - f(1)}{2 - 1}$

where $f(x) = x^3 + x - 1$.

So that means there exists a c so that $3 \cdot c^2 + 1 = \frac{[2^3 + 2 - 1] - [1^3 + 1 - 1]}{2 - 1}$

$\rightarrow 3c^2 + 1 = \frac{9 - 1}{1}$

$3c^2 + 1 = 8$

$-1 \quad -1$

$3c^2 = 7$

$\Rightarrow c^2 = 7/3$

$c = \pm\sqrt{7/3}$

for c to be between 1 and 2

$c = \sqrt{7/3}$