Limit Laws

Let f and g be continuous functions, let c be a constant, and n be a positive integer, then if each expression exists:

$$\lim_{x \to a} f(x) \pm g(x) = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x) \qquad \lim_{x \to a} cf(x) = c(\lim_{x \to a} f(x))$$

$$\lim_{x \to a} [f(x) \cdot g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x) \qquad \lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$$

$$\lim_{x \to a} [f(x)^n] = [\lim_{x \to a} f(x)]^n \qquad \lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)}$$

While working in a group make sure you:

- Expect to make mistakes but be sure to reflect/learn from them!
- Are civil and are aware of your impact on others.
- Assume and engage with the strongest argument while assuming best intent.

1. The temperature (in F) of coffee is approximated by $T(x) = 42 + 118e^{-0.04x}$ after x minutes. Find $\lim_{x \to 5^-} T(x)$ and interpret it's meaning.

2. Find $\lim_{x \to 7^-} \dfrac{3x}{\sqrt{x+2}}$

3. Find $\lim_{x \to 0^+} \dfrac{x}{\sqrt{x+4} - 2}$

4. Let $f(x) = 3x^2$. Either follow the steps below or compute the limit of the difference quotient of f at 2 directly, that is find:

$$\lim_{\Delta x \to 0} \frac{f(2 + \Delta x) - f(2)}{\Delta x}.$$

(a) Find $f(2)$.

(b) Find $f(2 + \Delta x)$, and simplify.

(c) Use (a) and (b) to find $\dfrac{f(2 + \Delta x) - f(2)}{\Delta x}$.

(d) Find $\lim_{\Delta x \to 0} \dfrac{f(2 + \Delta x) - f(2)}{\Delta x}$.

5. Let $g(x) = x^2 - 2x$. Either follow the steps below or compute the limit of the difference quotient of g at 3 directly, that is find:

$$\lim_{h \to 0} \frac{g(3 + h) - g(3)}{h}.$$

(a) Find $g(3 + h)$, and simplify.

(b) Use (a) to find $\dfrac{g(3 + h) - g(3)}{h}$.

(c) Find $\lim_{h \to 0} \dfrac{g(3 + h) - g(3)}{h}$.