(~~needs to~~ ~~to be~~ fixed)

~~typo~~ on back

Implicit Differentiation

Key

1. Assume that y is a function of x. Find $\frac{dy}{dx}$ in the following:

   (a) $x^3 + y^3 = 8$

$\frac{d}{dx}(x^3+y^3) = \frac{d}{dx}(8)$

$\frac{d}{dx}(x^3) + \frac{d}{dx}(y^3) = 0$

$3x^2 + \frac{d}{dx}(y^3) = 0$

$\frac{d}{dx}(y^3)$

$f(x) = x^3 \quad g(x) = y$

$f'(x) = 3x^2 \quad g'(x) = \frac{dy}{dx}$

$\frac{d}{dx}(y^3) = f'(y)\frac{dy}{dx} = 3y^2\frac{dy}{dx}$

$3x^2 + 3y^2\frac{dy}{dx} = 0$

$3y^2\frac{dy}{dx} = -3x^2$

$\frac{dy}{dx} = \frac{-3x^2}{3y^2} = \frac{-x^2}{y^2}$

   (b) $y = x^2y^3 + x^3y^2$

$\frac{d}{dx}(y) = \left[\frac{d}{dx}(x^2y^3)\right] + \left[\frac{d}{dx}(x^3y^2)\right]$

$\frac{dy}{dx} = \left[x^2\frac{d}{dx}(y^3) + \frac{d}{dx}(x^2)y^3\right] + \left[x^3\frac{d}{dx}(y^2) + \frac{d}{dx}(x^3)y^2\right]$

$\frac{dy}{dx} = x^2 3y^2\frac{dy}{dx} + 2xy^3 + x^3 2y\frac{dy}{dx} + 3x^2y^2$

$\frac{dy}{dx} - x^2 3y\frac{dy}{dx} - x^3 2y\left(\frac{dy}{dx}\right) = 2xy^3 + 3x^2y^2$

$\Rightarrow \frac{dy}{dx} = \frac{2xy^3+3x^2y^2}{1-3x^2y-2x^3y}$

   (c) $y = \sin(2x + 5y)$

$\frac{d}{dx}(y) = \frac{d}{dx}(\sin(2x+5y))$

$\frac{dy}{dx} = \cos(2x+5y)\left[2+5\frac{dy}{dx}\right]$

$\frac{dy}{dx} = 2\cos(2x+5y) + 5\frac{dy}{dx}\cos(2x+5y)$

$\frac{dy}{dx} - 5\frac{dy}{dx}\cos(2x+5y) = 2\cos(2x+5y)$

$\Rightarrow \frac{dy}{dx} = \frac{2\cos(2x+5y)}{1-5\cos(2x+5y)}$

   (d) $e^{xy} = e^{3x} - e^{4y}$

$\frac{d}{dx}(e^{xy}) = \frac{d}{dx}(e^{3x}) - \frac{d}{dx}(e^{4y})$

$f(x)=e^x \quad g(x)=xy \qquad f(x)=e^x \quad g(x)=3x \qquad F(x)=e^x \quad G(x)=4y$

$f'(x)=e^x \quad g'(x)=x\frac{dy}{dx}+y \qquad f'(x)=e^x \quad g'(x)=3 \qquad F'(x)=e^x \quad G'(x)=4\frac{dy}{dx}$

$e^{xy}\left(x\frac{dy}{dx}+y\right) = e^{3x}\cdot 3 - e^{4y}\cdot 4\frac{dy}{dx}$

$\frac{dy}{dx}xe^{xy} + ye^{xy} = 3e^{3x} - 4e^{4y}\frac{dy}{dx}$

$\frac{dy}{dx}xe^{xy} + 4e^{4y}\frac{dy}{dx} = 3e^{3x} - ye^{xy}$

$\frac{dy}{dx} = \frac{3e^{3x}-ye^{xy}}{xe^{xy}+4e^{4y}}$