

Show *all* your work (numerically, algebraically, or geometrically) for the following problems. Supporting work is needed to earn credit.

1. The work for the following problems is *wrong*. Explain why the solution is wrong *and* then find the correct solution (numerically, graphically, or algebraically).

(a) [5] Find $\frac{dy}{dx}$ given $y = \log_2(x^2(x^4 + 2x^2))$.

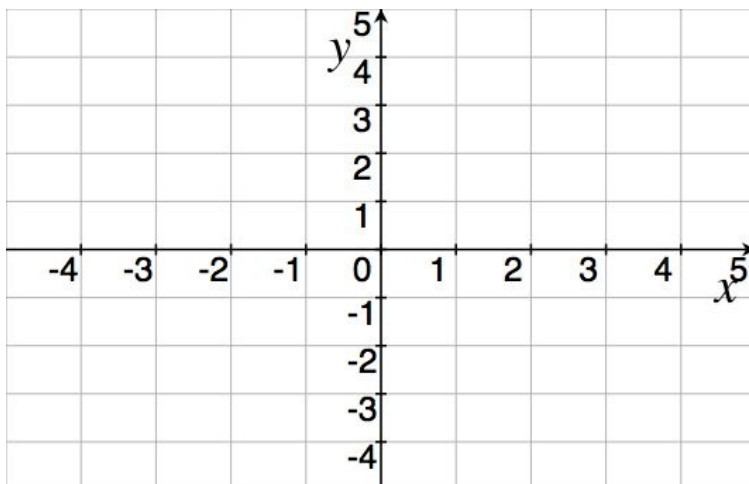
$$\begin{aligned} y &= \log_2((x^2)(x^4 + 2x^2)) \\ &= \log_2(x^6 + 2x^4) \\ \frac{dy}{dx} &= \frac{1}{x \ln(2)}(x^6 + 2x^4) + \log_2 \cdot (6x^5 + 8x^3) \end{aligned}$$

(b) [4] Find $\frac{dy}{dx}$ given $\lim_{\theta \rightarrow \frac{\pi}{2}^-} \frac{\sin(2\theta)}{1 - \cos(2\theta)}$

$$\begin{aligned} \lim_{\theta \rightarrow \frac{\pi}{2}^-} \frac{\sin(2\theta)}{1 - \cos(2\theta)} &=_{L'H} \lim_{\theta \rightarrow \frac{\pi}{2}^-} \frac{\cos 2\theta \cdot 2}{0 - (-\sin 2\theta \cdot 2)} \\ &= \lim_{\theta \rightarrow \frac{\pi}{2}^-} \frac{\cos 4\theta}{\sin 4\theta} \\ &= \frac{\cos 2\pi}{\lim_{\theta \rightarrow \frac{\pi}{2}^-} \sin 4\theta} = \frac{1}{0^-} = -\infty \end{aligned}$$

2. [5] Draw a graph for a function $\alpha(x)$, that satisfies *all* of the following:

- (a) α is continuous on the interval $(-2, 4)$,
 (b) $\lim_{x \rightarrow -2} \alpha(x) = \infty$,
 (c) $\alpha'(2) = -1$, and
 (d) $\lim_{x \rightarrow 4} \alpha(x) \neq \alpha(4)$.



3. The following graph is a function, d , that returns the *distance* (in feet) a fly is from a spider web after t seconds.

(a) [3] Is the fly ever 1 foot away?
If so, when?

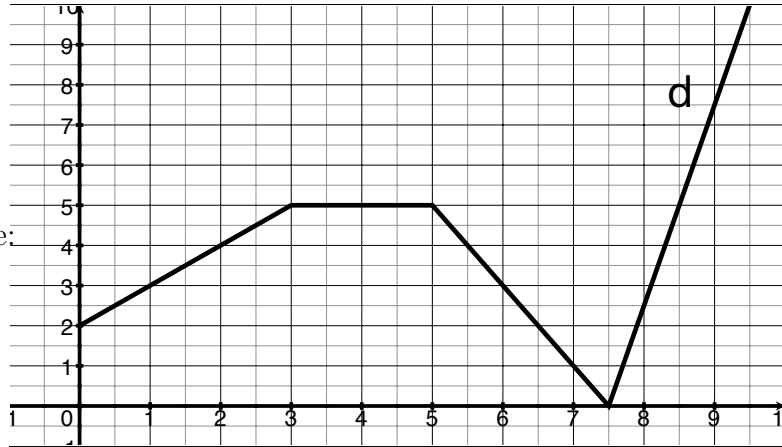
(b) [5] Estimate the following, if possible:

$$\lim_{t \rightarrow 6} \left(\frac{d(t)}{3} - 2 \right)$$

$$d(2)$$

$$\frac{d}{dt}d|_{t=2}$$

(c) [3] What is the speed of the fly when $t = 2$ and is the fly moving towards or away from the web?



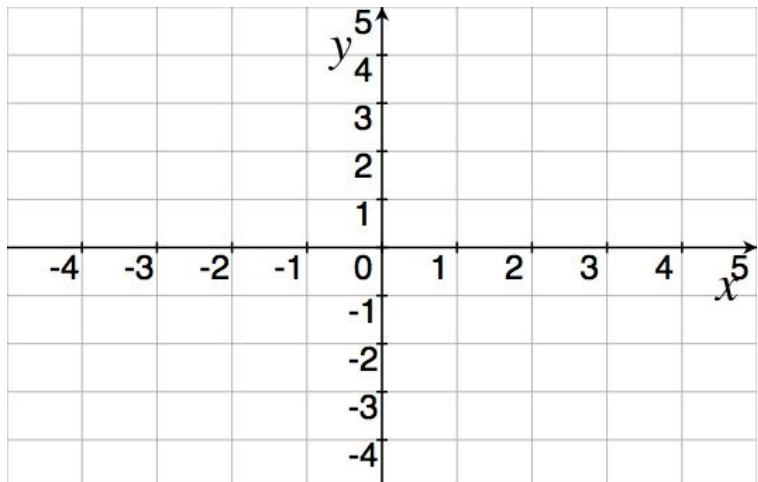
4. [5] Draw a graph for a function $\beta(x)$, that satisfies all of the following:

(a) $\lim_{x \rightarrow -\infty} \beta(x) = 2$,

(b) β is continuous on the interval $(-2, 3)$,

(c) $\beta'(-1)$ does not exist, &

(d) $\beta'(x) < 0$ when $0 < x$.



5. Let f be the function graphed on the right. Let $h(x) = 2^x$. Using this and the information about g given below, find the following (if possible!):

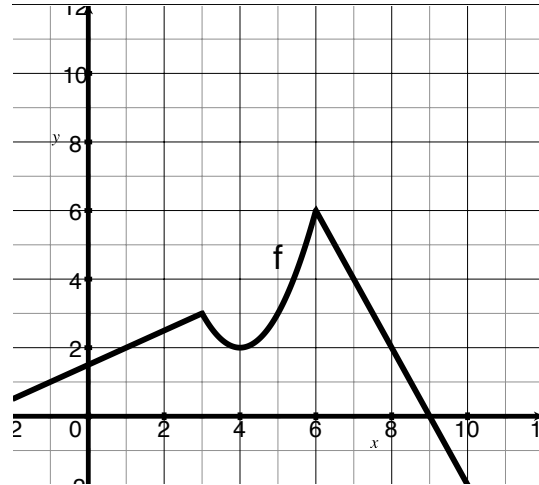
$$g(2) = 1 \quad g'(2) = -6$$

$$g(9) = 0 \quad g'(9) = 5$$

- (a) [2] Estimate all x so that $f(x) = 1$.

- (b) [2] Estimate all x so that $f'(x) = 0$.

- (c) [3] Find $(f \cdot h)'(1)$.



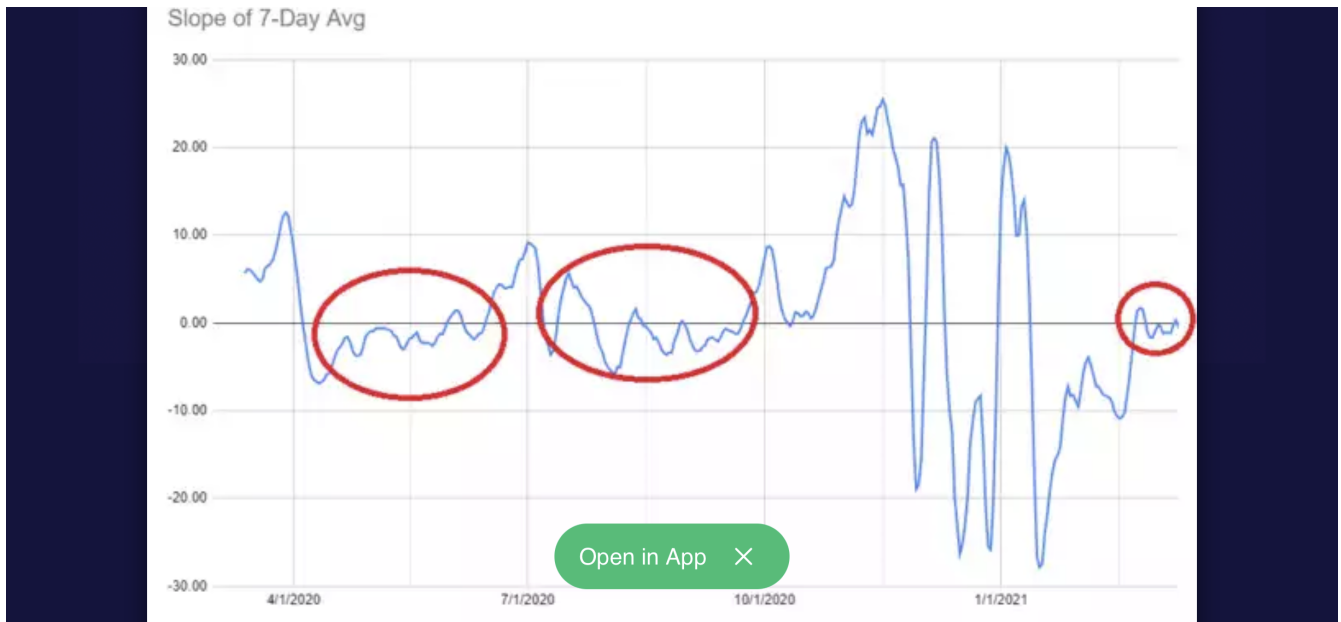
- (d) [3] Find $\frac{d}{dx}(g(h(x)))|_{x=1}$.

- (e) [3] Find $\lim_{x \rightarrow 9} \frac{f(x)}{g(x)}$

- (f) [2] Find the equation of the line tangent to g at $x = 9$.

- (g) [2] Identify the area on the graph described by $\int_6^{10} f(x) dx$.

- (h) [2] Find the value of $\int_6^{10} f(x) dx$.



6. Use the graph provided by JCRooks on CoronavirusWA Reddit on Mar 13th duplicated above for the following questions. JCRooks is plotting the SLOPE of the 7-day average of new Covid-19 cases in King County over time.
- (a) [2] Describe what is happening to the 7-day average of new Covid-19 cases in King County at the end of December.
- (b) [3] Identify a time that the 7-day average of new Covid-19 cases was at a local minimum. Explain/justify your answer.
- (c) [2] Why do you think JCRooks provided this graph of the *slope* instead of the 7-Day average of new Covid-19 cases directly?

7. [4] Find $\frac{dy}{dx}$ where $y = \frac{x^{0.5x}}{\arctan(x)}$

8. Ryan and Stella were being chased by a pack of zombies. At point P they decided to split up and Ryan ran north at 12 ft/s. Stella waited for five seconds to try to draw most of the zombies towards her and then started to run east at 15 ft/s.

- (a) [3] Find an equation relating the speed that Ryan and Stella are moving apart to other variables.
- (b) [2] At what rate are Ryan and Stella moving apart two minutes after Stella started running (assuming they are still alive!)

9. An industrial tank is formed by adjoining two hemispheres to the ends of a right circular cylinder. The total volume of the tank is 5000 cubic feet. The hemispherical ends cost \$14 per square foot where as the sides cost only \$8 per square foot.
- (a) [4] Find a function that describes the cost of the tank dependent only on one variable.
 - (b) [2] Outline the steps needed to find the maximum enclosed area. Do *not* perform the steps!!!
10. [4] Choose a problem from this exam that you've already answered,
- (a) show a second way of approaching/building a solution, and
 - (b) explain why you did not choose this second method initially.