

TMATH 120 Algebra Review Sheet

Proficiency of the following topics is assumed for TMATH 120. Consider doing one of the “You Try” problems from each of the sections below (and check your answer in the back of the book). If you feel the need to practice more, read the given examples carefully, complete the remaining “You Try” problems, and choose additional odd problems from the same section of the text until you are comfortable.

Simplification: You Try: §A.1#59, 63, 65, 67, 81, 83, 85, 99

Apply order of operations to numerical and algebraic expressions; use the rules for exponents.

Ex 1. Simplify: $-3^2 + \left(\frac{3}{2}\right)^2 + \left(\frac{1}{6}\right)^{-1} + (-2)^3$.

Solution:

$$-3^2 + \left(\frac{3}{2}\right)^2 + \left(\frac{1}{6}\right)^{-1} + (-2)^3 = -9 + \frac{9}{4} + \frac{6}{1} + -8 = -17 + \frac{9}{4} + \frac{6}{1} = \frac{-68}{4} + \frac{9}{4} + \frac{24}{4} = \frac{-35}{4}$$

(Another answer is -8.75 . $-8\frac{3}{4}$ is also technically correct, but we avoid mixed fractions because of potential confusion: Taken out of context, $-8\frac{3}{4}$ could mean *either* $-(8 + \frac{3}{4})$ *or* $(-8) \cdot \frac{3}{4}$.

Algebraic Manipulation: You Try: §A.2#9, 13, 15, 17, 19, 23, 37

Add, subtract, multiply, and divide algebraic expressions; combine alike algebraic terms; apply the distributive property to algebraic expressions (the term “FOIL” is often used when distributing between two binomials).

Ex 2 Simplify by combining like terms: $(9x^2 + 4xy - 7y^2) - (5xy - 6x^2 - 1)$.

Solution:

$$(9x^2 + 4xy - 7y^2) - (5xy - 6x^2 - 1) = 9x^2 + 4xy - 7y^2 - 5xy + 6x^2 + 1 = 15x^2 - xy - 7y^2 + 1$$

Ex 3 Expand and simplify completely: $3(4v - 2a)^2$.

Solution:

$$\begin{aligned} 3(4v - 2a)^2 &= 3(4v - 2a) \cdot (4v - 2a) = 3[(4v)(4v) - (4v)(2a) - (2a)(4v) + (2a)(2a)] \\ &= 3[16v^2 - 16av + 4a^2] = 48v^2 - 48av + 12a^2 \end{aligned}$$

Factoring: You Try: §A.2#97, 101, 105, 107

Factor binomials of the form $a^2 - b^2$; factor trinomials of the form $ax^2 + bx + c$ using factoring by grouping or trial and error; recognize and factor out the greatest common factor of an expression; check answers by using distribution.

Ex 5 Factor completely: $16w^4 - 81$.

Solution:

$$16w^4 - 81 = (4w^2 + 9)(4w^2 - 9) = (4w^2 + 9)(2w + 3)(2w - 3)$$

(Note that $4x^2 + 9$ does not factor any further since the *sum* of squares is prime.)

Ex 6 Factor completely: $12t^2 + 14t - 6$.

Solution:

$$12t^2 + 14t - 6 = 2(6t^2 + 7t - 3) = 2(2t + 3)(3t - 1)$$

Equations of Lines: You Try: §1.2#1, 9, 11, 15, 17

Find the slope of the line defined by two points; write the equation of a line in slope-intercept form.

Ex 4. Find the slope of the line passing through the points $(-2, 7)$ and $(4, -7)$.

$$\text{Solution: slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7 - 7}{4 - (-2)} = \frac{-14}{6} = \frac{-7}{3}$$

Rational Expressions: You Try: §A.3#3, 5, 9, 11, 15, 17, 21, 29, 37, 45

Simplify fractions by finding factors in common; add/subtract fractions by first rewriting with the least common denominator; multiply/divide rational expressions; simplify complex fractions.

Ex 7 Reduce to lowest terms: $\frac{10x - 20}{2x^2 - 8}$.

$$\text{Solution: } \frac{10x - 20}{2x^2 - 8} = \frac{10(x - 2)}{2(x^2 - 4)} = \frac{10(x - 2)}{2(x + 2)(x - 2)} = \frac{2(x - 2)}{2(x - 2)} \cdot \frac{5}{x + 2} = \frac{5}{x + 2}$$

Ex 8 Perform the indicated operations and simplify: $\frac{2}{3x + 2} - \frac{1}{3x + 1}$.

$$\begin{aligned} \text{Solution: } \frac{2}{3x + 2} - \frac{1}{3x + 1} &= \frac{3x + 1}{3x + 1} \cdot \frac{2}{3x + 2} - \frac{1}{3x + 1} \cdot \frac{3x + 2}{3x + 2} \\ &= \frac{2(3x + 1)}{(3x + 1)(3x + 2)} - \frac{3x + 2}{(3x + 1)(3x + 2)} = \frac{3x}{(3x + 1)(3x + 2)} \end{aligned}$$

(There is no further simplification here, $3x$ has *no* factors in common with $3x + 1$ or $3x + 2$.)

Ex 9 Perform the indicated operations and simplify: $\frac{2z + 6}{12z} \div \frac{z^2 - 9}{9z^3 + 18z^2}$.

$$\begin{aligned} \text{Solution: } \frac{2z + 6}{12z} \div \frac{z^2 - 9}{9z^3 + 18z^2} &= \frac{2z + 6}{12z} \cdot \frac{9z^3 + 18z^2}{z^2 - 9} = \frac{2(z + 3)}{12z} \cdot \frac{9z^2(z + 2)}{(z + 3)(z - 3)} \\ &= \frac{2 \cdot 3 \cdot z(z + 3)}{2 \cdot 3 \cdot z(z + 3)} \cdot \frac{3z(z + 2)}{2(z - 3)} = \frac{3z(z + 2)}{2(z - 3)} \end{aligned}$$

Solving Equations: You Try: §A.6#1, 5, 7, 11, 15, 17, 31, 35

Solve linear equations; solve quadratic equations by employing factoring or the quadratic formula.

Ex 10 Solve for m : $3(m + 4) + 2m = 4 - 3m$.

Solution

$$3(m + 4) + 2m = 4 - 3m \Rightarrow 3m + 12 + 2m = 4 - 3m \Rightarrow 8m = -8 \Rightarrow m = -1$$

Ex 11 Solve for q : $2q^2 + 7 = 9q$.

Solution

$$2q^2 + 7 = 9q \Rightarrow 2q^2 - 9q + 7 = 0 \Rightarrow (2q - 7)(q - 1) = 0 \Rightarrow q = \frac{7}{2} \text{ or } q = 1$$