

Some More Word Problems

Next week there will be a chance to earn extra credit for the exam. Groups of two or three can choose one of the numbered problems below and present it to the class. Each group can earn up to 4% extra credit for the next exam where each earned point comes from the criteria following.

- Mastery of the problem: Do you understand the problem? Do you understand all of the steps in solving the problem? Would you be able to solve the problem if given a slightly different question?
- Presentation of the problem: You are presenting *new* material to your classmates that will be on their exam next week. Your peers need this time to be *taught* the material (not merely shown an example). Take care to explain the steps you take and why you take them.
- Presentation: Do all group members speak (needed for everyone to get the extra credit!)? Did you anticipate questions from the audience?
- Fielding questions: Can you understand someone's question about the material and formulate a cohesive answer? Note that these will be posted in the threaded discussion so you will have to monitor your post and make sure to answer questions in a timely fashion.

As always, while working in a group make sure you:

- Expect to make mistakes but be sure to reflect/learn from them!
 - Are civil and are aware of your impact on others.
 - Assume and engage with the strongest argument while assuming best intent.
1. An advertisement for real estate published in the 26 July 2004 electronic edition of the New York Times states:

Did you know that the percent increase of the value of a home in Manhattan between the years 1950 and 2000 was 721%? Buy a home in Manhattan and invest in your future.

Suppose instead of buying a home in Manhattan in 1950, someone had invested money in a bank account that compounds interest once per month. What annual interest rate would he bank have to pay to equal the growth claimed in the above ad?
 2. Browsing cars in a car-lot you see a sign in the window of a \$15,000 car that offers you a choice between two deals. \$1,000 back now (which you would use towards the principal) or 1.5% reduced interest rate for the first year. The standard rate for a loan between \$12,000 and \$20,000 is 5.29% for 5 years compounded continuously. If you plan to not make any payments for the first two years, which plan is a better choice?
 3. Entropy S is a function of the number of possible states W , that are accessible to a system with a given amount of energy. We can explicitly compute entropy by $S = k \ln(W)$ where k is Boltzmann's constant. If liquid A has 100,000 times more possible states than liquid B, which liquid has a higher entropy and by how much more?

4. You have four ten-year subsidized loans you took out to pay for college. Below is a table of the loans taken and their respective effective annual interest rates (AIR):

loan (\$)	8,000	9,000	10,000	12,000
AIR (%)	3.51	4.22	5.01	6.31

After graduation you are given the option of consolidating (that is take out one loan to pay off *all* the balances on your current loans). Assume all loans are continuously compounded once interest start accruing. You do not have a job lined up yet so you doubt you will be able to make any payments for the three years, what rate would you need to consolidate your loans at to be in a better position three years from now?

5. Chad just graduated but does not have a job lined up yet. In his last year of graduate school he made ends meet by using his credit card and now has a balance of \$7,000. His credit card compounds monthly with an annual interest rate of 19.9%. Assume the worst and that Chad won't find a job for the next two years and will have to move in with his parents. He won't be able to make any payments on his credit card bill.

Just before graduation, Chad received an ad for a State Farm Good Neighbor Visa Credit Card. The card will transfer his balance (with a 2.90% balance transfer fee) and then give him a lower annual interest rate of 10.24% that is still compounded monthly. Should he take this option instead of staying with his first credit card?

6. In 2009, according to PayDay Loan's website, a Californian take up to a \$255 loan for 31 days for a one-time 18% charge. If you are in California and you use PayDay Loans to take out a \$100 loan for a year (that is, at the end of 31 days, you have to take out another PayDay Loan to pay off the previous PayDay Loan) and may need to take out two PayDayLoans to pay off the previous one.

(a) How much money will you owe by the end of the year?

(b) What is the effective annual interest rate?

7. According to the University of Texas, the total amount of habitable land on earth is 24,642,757 square miles. Recall 1 mile equals 5,280 feet. According to the World Population Clock (www.worldometers.info/world-population/), the earth's population is currently about 8.01 billion people and it is increasing with an exponential rate of growth of 1.1% (with respect to the growth $P(1 + r)^t$ model. When will there be standing room only? (Assume each person needs about 4ft^2 to stand.)

8. Some paper taken from the Dead Sea Scrolls was found to have a carbon-14 to carbon-12 ratio of .795 times that found in living plants. Given that the half-life of carbon-14 is 5,730 years and that the test was done in 1955, estimate the age of the scroll.

9. When an underwater light beam is turned on in sea water, the resulting intensity of the light decays exponentially along the direction of the beam. In fact, in clear sea water, the approximate percentage of the original intensity x meters from the source measured along the beam is given by intensity $I = 100e^{-1.4x}$

- (a) Find the intensity at a depth of 2 meters.
 - (b) Use the formula given above to help explain why in any ocean almost no plants can grow at a depth of more than 10 meters.
10. *Newton's Law of Cooling:* Given the initial temperature of an object T_0 and the surrounding temperature T_s we can find the temperature T of the object at time t by

$$T(t) = T_s + (T_0 - T_s)e^{-kt},$$

where k is a positive constant that depends on the object.

Dr. Card finds that tea burns his tongue if it is above 140°F and tastes bad if it is below 105°F . Initially Dr. Card used the paper cups provided by Metro that have a constant $k = 0.05$, but then he bought his own tea mug that has a constant $k = 0.03$ (we are of course referring to the k that shows up in Newton's Law of cooling).

- (a) Which container does a better job keeping the coffee warm? How does knowing the values of k allow you to figure out the answer?
 - (b) If Dr. Card gets a cup of coffee using his own mug that is 160° at 8:30am and the outside temperature is 42°F , how long should he wait to drink the coffee so that he does not burn his mouth?
 - (c) How long does Dr. Card have to drink the coffee before it starts to taste bad if he uses the paper cup? What if he uses his own mug?
11. *Brightness:* Given two stars of magnitudes m_1 and m_2 have apparent brightness b_1 and b_2 respectively, then

$$m_2 - m_1 = 2.5 \log \left(\frac{b_1}{b_2} \right)$$

Find the magnitude m of a star that is 650 times as bright as one of magnitude 7.25.

12. Create your own problem. You must have this approved by the instructor.