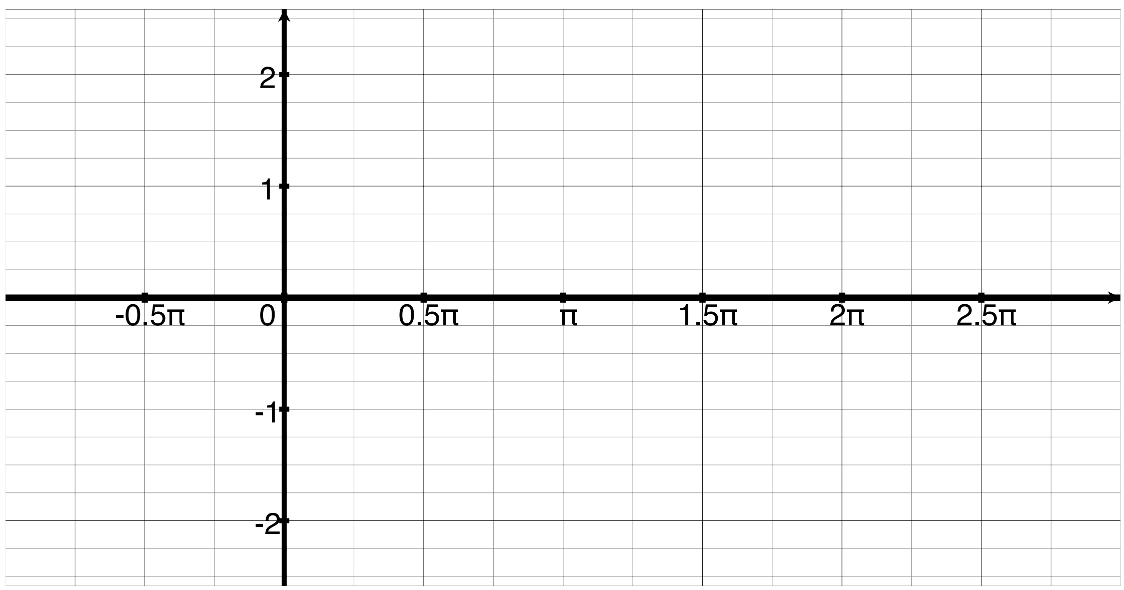
Trigonometric Graph Transformations

While working in a group make sure you:

- Expect to make mistakes but be sure to reflect/learn from them!
- Are civil and are aware of your impact on others.
- Assume and engage with the strongest argument while assuming best intent.

1. Let $f(\theta) = \cos\theta$. Graph f on the axis provided.



2. Recall Section §1.5 (what was that about again?) and finish the following sentence: The graph of $g(\theta) = 2\cos\left(\theta + \frac{\pi}{2}\right)$ looks like the graph of f, but...

3. Draw the graph of g on the axis above.

4. The *amplitude* is (often defined as) half the difference in y values between the "peak" to the "trough". Identify the amplitude of f and g.

5. Functions are called periodic if the function "repeat themselves". That is F is periodic if $F(\theta) = F(\theta + k)$. The smallest such constant k for a function is the period.

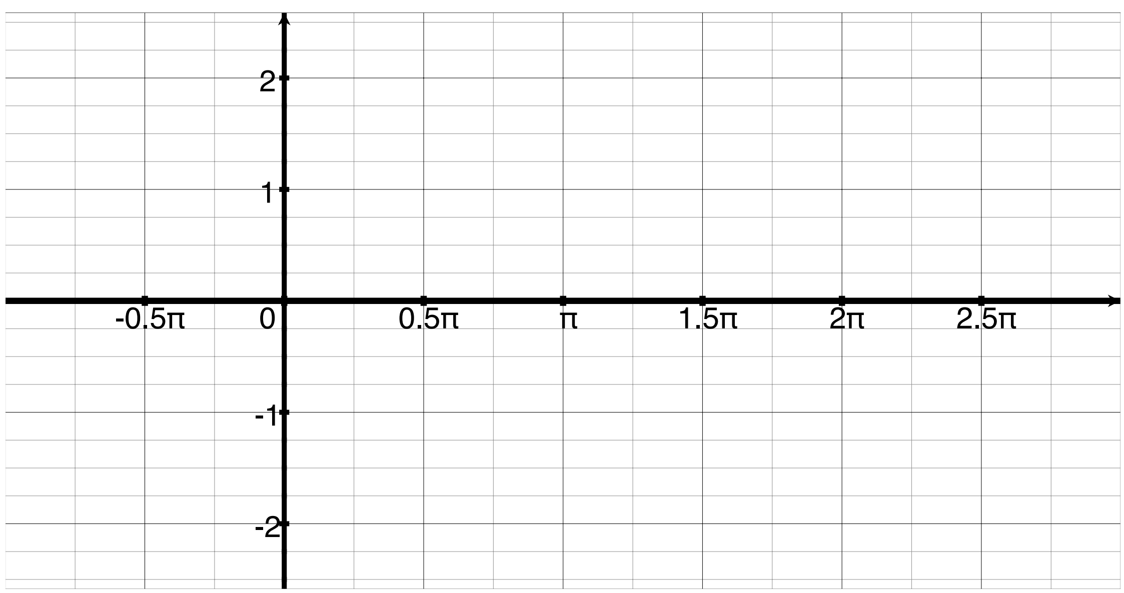
   - Find the period of $\cos\theta$?

   - Find the period of $\sin\theta$?

6. Notice the above functions make a complete wave in their respective periods. Think of θ as time, then 2θ is rather like time running twice as fast. Thinking in these terms we see that $\cos 2\theta$ completes its wave in half the time. Thus instead of taking 2π to finish a period, $\cos 2\theta$ completes the wave in $\frac{2\pi}{2}$ or π.

   - What is the period of $\sin(2\theta)$?

7. Note #6 implies the graph of $\sin 2\theta$ looks like the graph of $\sin\theta$ but horizontally stretched by a factor of $\frac{1}{2}$ (or compressed by a factor of two). We could also say that the x coordinate is divided by 2. Draw $\sin(2\theta)$ by either plotting points, using technology, or reasoning.



8. Try to use the above reasonings to find the periods for the following functions:

   - $\sin(3\theta)$?
   - $3\cos(\frac{1}{2}\theta)$?

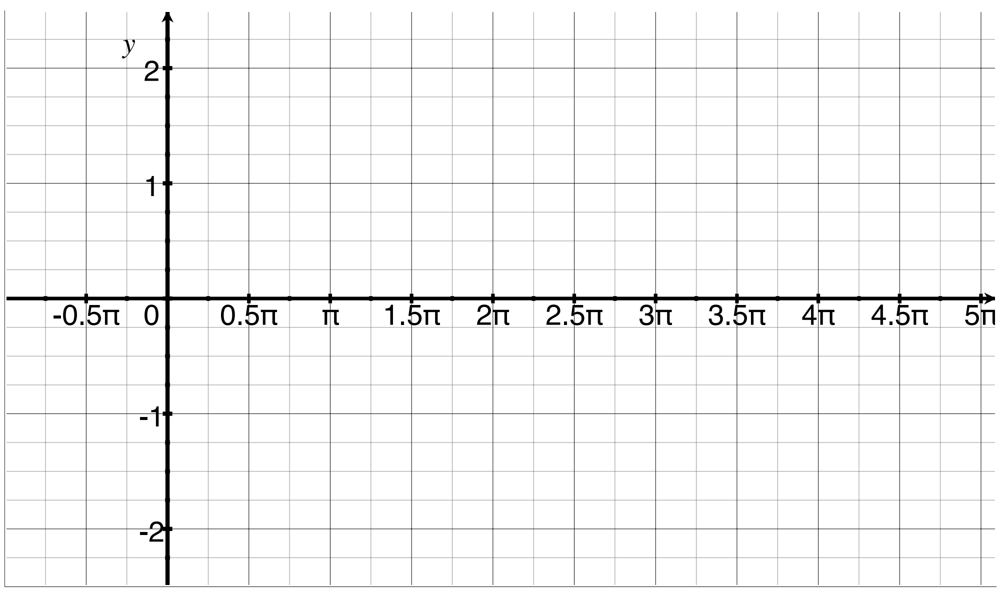
9. Can you find the amplitudes for the functions in #8 without graphing them?

Verify #8 and 9's answers by looking at Examples 3 & 4 on pages 325 & 326 respectively.

10. Draw the graph of $y = a\sin(bx)$ where $a > 0$ and $b > 0$.
Consult the box on page 326 to check your answer.

11. Use your results from above to find the amplitude and period for $-1.5\sin\left(\frac{1}{2}\theta\right)$

12. Use the information you gathered in #10, to draw $-1.5\sin\left(\frac{1}{2}\theta\right)$ below.



Horizontal Stretches meet Horizontal Shifts

1. Consider $\cos(2\theta + \pi)$. The graph of this could look like the graph of $\cos\theta$ but...

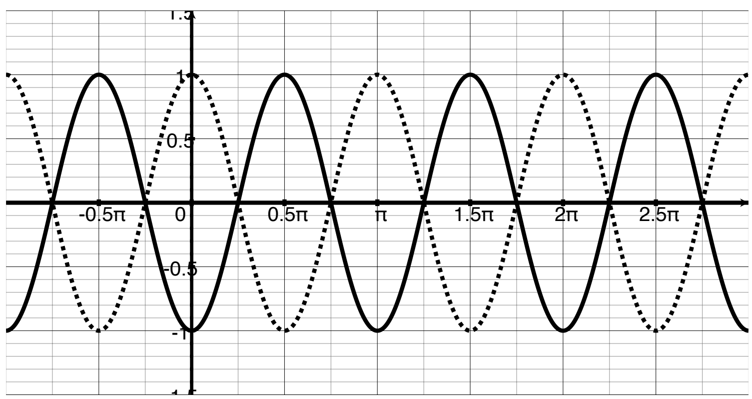
   1) shifted left π units, and then,
   2) horizontally stretched by $\frac{1}{2}$ so that the curve has period π

   or

   1) horizontally stretched by $\frac{1}{2}$ so that the curve has period π, and then
   2) shifted left π units

   The two process above do *not* yield the same curve. In fact, the solid curve results from following the directions on the left and the dotted curve results from following the directions on the right.

   Which one is the graph of $\cos(2\theta + \pi)$?



2. Customarily (in computer engineering, and physics), people write $\cos(2\theta + \pi)$ as $\cos(2(\theta + \frac{\pi}{2}))$. The number $-\frac{\pi}{2}$ is called the phase shift. Explain why the number $-\frac{\pi}{2}$ would be called "phase shift".

3. Use the above to find the amplitude, period, and phase shift of $\frac{3}{4}\cos\left(2\theta + \frac{2\pi}{3}\right)$.