MIDTERM 1 TQS 120 Practice

NAME: This is a sample midterm to be used for practice. This is *not* a template for the midterm that will be given in class. Many of the questions on the Midterm will look quite different than those appearing here.

1. [4] TRUE/FALSE: Circle T in each of the following cases if the statement is *always* true. Otherwise, circle F. Let $f$ be a function, and $x$, $y$, and $z$ be real numbers with $z \neq 0$.

   T F $\frac{3x+y}{3z} = \frac{x+y}{z}$

   T F $(x+y)^2 = x^2 + y^2$ $\quad (x+y)^2 = (x+y)(x+y) = x^2+2xy+y^2$

   T F $|x| = x$ $\quad$ ex $|-1| \neq 1$

   T F The function $\sqrt{(x-\sqrt{2})}$ has the domain $[\sqrt{2}, \infty)$ $\quad x-\sqrt{2} \geq 0$, $x \geq \sqrt{2}$

   Show your work for the following problems. The correct answer with no supporting work will receive NO credit (this includes multiple choice questions).

2. [3] Given $\frac{1}{r} + \frac{1}{t} = \frac{1}{s} + \frac{1}{u}$, solve for r.

$$\frac{1}{r} = \frac{1}{s} + \frac{1}{u} - \frac{1}{t}$$

$$sut \quad \frac{1}{r} = \frac{ut+st-su}{sut} \quad , sut$$

$$r \cdot \frac{sut}{r} = ut+st-su \quad \cdot r$$

$$sut = (ut+st-su)r$$

$$r = \frac{sut}{ut+st-su}$$

3. [4] Let the following describe the function $\alpha$:

| input: | $\bigcirc$ | $\star$ | $\Delta$ | $\star + \Delta$ |
|---|---|---|---|---|
| output: | 4 | -2 | 3 | -4 |

Find the following if possible:

$\alpha(\star) + \alpha(\Delta)$

$-2 + 3 = 1$

$\alpha(\star + \Delta)$

$-4$

$\alpha(\bigcirc) \times \alpha(\star + \Delta)$

$4 \cdot -4 = -16$

$\alpha(\Delta + \Delta)$

? not defined ?

4. Consider $f(x) = \frac{x-1}{x}$ and $g(x) = 3x - 4$.

- [2] What is $f(z + \sqrt{2})$? Do *not* expand this.

$$f(z+\sqrt{2}) = \frac{z+\sqrt{2}-1}{z+\sqrt{2}}$$

- [3] Find the rule for $f \circ g$ and *simplify* as much as possible.

$$(f\circ g)(x) = f(g(x)) = f(3x-4) = \frac{3x-4-1}{3x-4} = \frac{3x-5}{3x-4}$$

- [2] The function $f$ is one-to-one, find it's inverse.

Step 1 $f(y) = \frac{y-1}{y}$

Step 2 $x = \frac{y-1}{y}$

Step 3 $xy = y-1$

$xy - y = -1$

$y(x-1) = -1$

$y = \frac{-1}{x-1}$

5. [4] Consider the points $P = (3, 4)$ and $Q = (-1, -2)$. Find the equation to a line that goes through the point $(1, 1)$ and has a perpendicular slope to the line connecting $P$ and $Q$.

Slope of $\overline{PQ} = \frac{4+2}{3+1} = \frac{6}{4} = \frac{3}{2}$

Slope $\perp$ to $\overline{PQ}$ is $-2/3$

so $y = -2/3\, x + b$

b/c passes through (1, 1)

$1 = -2/3(1) + b$

$1 + 2/3 = b$

$5/3 = b$

so $y = -2/3\, x + 5/3$

- [1] What is the $y$ intercept of the line you found?

when $x = 0$ so

$-2/3 \cdot 0 + 5/3 = 5/3$

- [1] What is the $x$ intercept of the line you found?

when $y = 0$ so

$0 = -2/3\, x + 5/3$

$-5/3 = -2/3\, x$

$-5/3 \cdot -3/2 = x$

$5/2 = x$

6. [4] Find the domain of $f$ where $f(x) = \frac{2-\sqrt{5-2x}}{x+10}$.

sq roots only take #'s greater or equal to zero And Den $\neq 0$

$5 - 2x \geq 0$

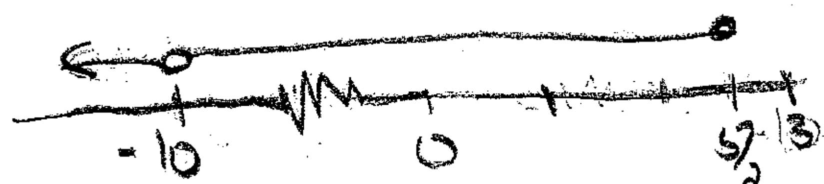
$-2x \geq -5$

$x \leq 5/2$

$x + 10 \neq 0$

$x \neq -10$

$x \neq -10$




(13)

$(-\infty, -10) \cup (-10, 5/2]$ so

3

7. [4] Given that $f(x) = x^2 - 5x - 6$. Write $f$ in vertex form.

$f(x) = x^2 - 5x - 6$

$+(5/2)^2 \qquad +(5/2)^2$

$f(x) + 25/4 = \underbrace{x^2 - 5x + \tfrac{25}{4}} - 6$

$f(x) + 25/4 = \underbrace{(x - 5/2)^2} - 6$

$\rightarrow f(x) = (x - 5/2)^2 - 6 - \frac{25}{4}$

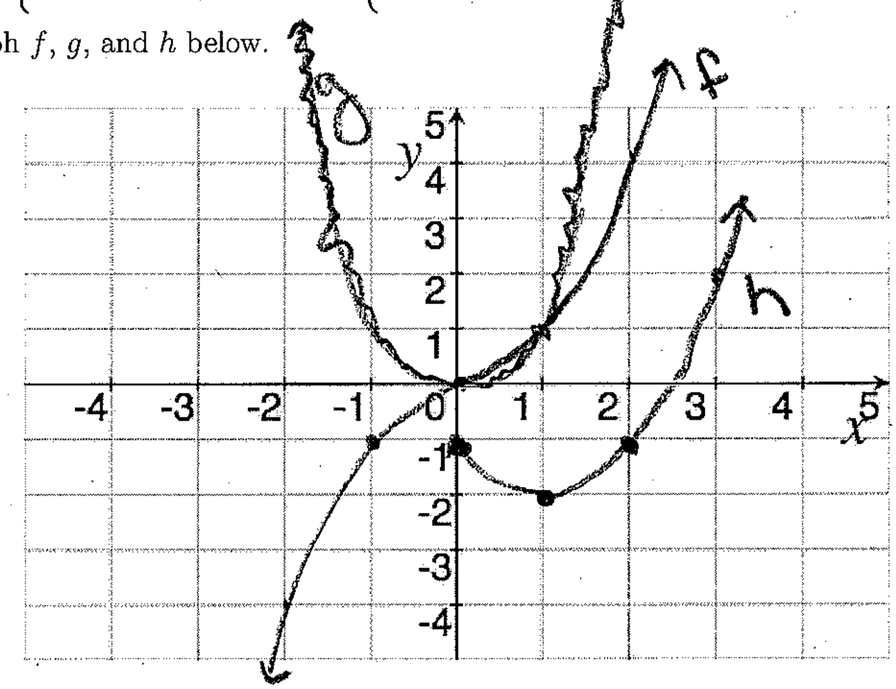
$= (x - 5/2)^2 - \frac{24}{4} - \frac{25}{4}$

$= (x - 5/2)^2 - \frac{53}{4}$

8. Given that the *functions* $f$, $g$, and $h$ are defined by:

$$f(x) = \begin{cases} x^2 & \text{if } 0 \geq x \\ -x^2 & \text{if } x < 0 \end{cases} \quad g(x) = \begin{cases} x^3 & \text{if } 0 \geq x \\ -x^3 & \text{if } x < 0 \end{cases} \quad h(x) = (x-1)^2 - 2$$

[3] Graph $f$, $g$, and $h$ below.




[3] Identify each function above as even, odd, neither, or both.

f is odd

g is even

h is neither

9. [4] Simplify the following as much as possible:

$$\frac{(2x^4y^3)^3(6xy^3)^{-3}}{x^4y^4}$$

$$\frac{2^3(x^4)^3(y^3)^3\,6^{-3}x^{-3}(y^3)^{-3}}{x^4y^4} = \frac{8x^{12}y^9y^{-9}}{x^4y^4\,6^3x^3} = \frac{8x^{12}y^{9-9}}{x^{4+3}6^3y^4} = \frac{8x^{12}}{6^3x^7y^4}$$

So $\frac{x^5}{27y^4}$

10. [3] Find a cubic polynomial whose graph passes through the points $(-2, 0)$ and $(1, 0)$ and has a root at 6. Note: there are many correct answers possible here.

cubic w/ roots $-2, 1$ & $6$

so

$(x-2)(x-1)(x-6)$

$(x+2)(x-1)(x-6)$ works

11. The height $y$ (in feet) of a ball thrown by a child on the planet Gethen is

$$y = -x^2 + 15x + 3$$

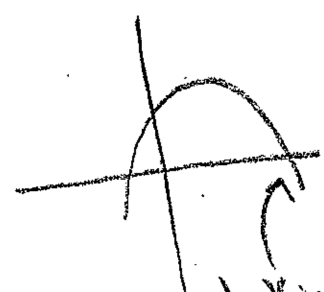
where $x$ is the horizontal distance in feet from the point at which the ball is thrown. Answer the following questions.

(a) [2] How high is the ball when it leaves the child's hand?

when $x$ is $0$

$-0^2 + 15\cdot 0 + 3 = 3\text{ ft}$

(b) [2] How far from the child does the ball hit the ground?



looking for this coordinant

looking for a root so $x$ value when

$0 = -x^2 + 15x + 3$

$\Rightarrow x = \frac{-15 \pm \sqrt{15^2 - 4(-1)(3)}}{2(-1)} = \frac{-15 \pm \sqrt{237}}{-2}$

$= \frac{-15 \pm 15.39}{-2} = 15.192\text{ ft}$ or ~~$-.195\text{ ft}$~~