

Polynomials

While working in a group make sure you:

- Expect to make mistakes but be sure to reflect/learn from them!
- Are civil and are aware of your impact on others.
- Assume and engage with the strongest argument while assuming best intent.

Definition 0.1. A *polynomial function of degree n* is a function of the form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$$

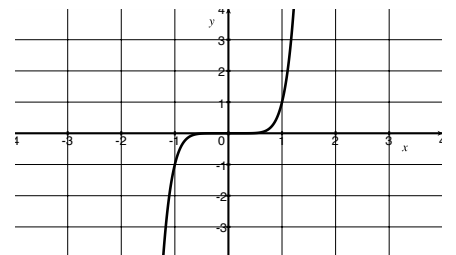
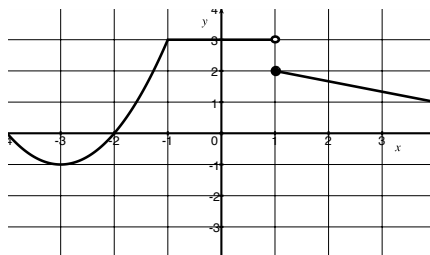
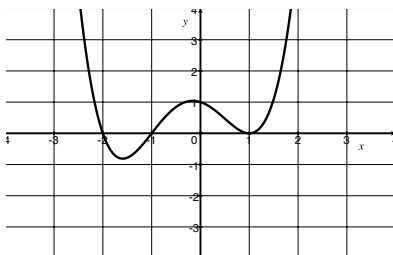
where n is a nonnegative integer and the coefficients $a_n, a_{n-1}, \dots, a_2, a_1, a_0$ are real numbers with $a_n \neq 0$. The term $a_n x^n$ is called the *leading term*, the number a_n is called the *leading coefficient*, and a_0 is the *constant term*.

1. For each of the expressions below, determine if it is a polynomial and *if it is*, determine the degree:

expression	polynomial? (yes/no)	leading term (if applicable)	degree (if applicable)
$-117x^4 + 6x^{12} + x$			
$2^x - 5x^2$			
$\sqrt{5}x^2 - \pi$			
$7x^8 - 4.56x^4 - 7x^8 + x^2$			
3			
0			

Note that we have already spent a few days on first degree and second degree polynomials (i.e. lines & parabolas). We now turn to higher order polynomials.

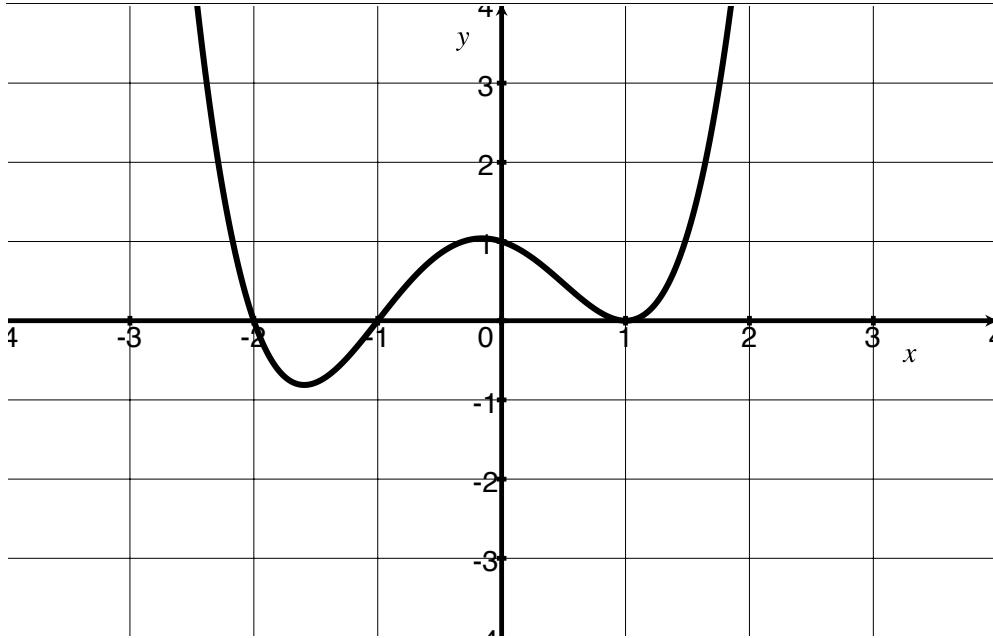
2. Which of the following could be a polynomial?



3. For each graph above that could be a polynomial, determine what degree it could have.

Definition 0.2. A number c is called a *zero* of a function p if $p(c) = 0$. This number is also called a *root* or an *x-intercept*.

4. Given the graph below of a polynomial of degree four, find the algebraic rule.



Definition 0.3. A *rational function* r is a function of the form $r(x) = \frac{f(x)}{g(x)}$ where f and g are polynomials with g not equal to the zero function.

5. Perform the division: $\frac{2x^4 + x^3 - 16x^2 + 18}{x + 2}$