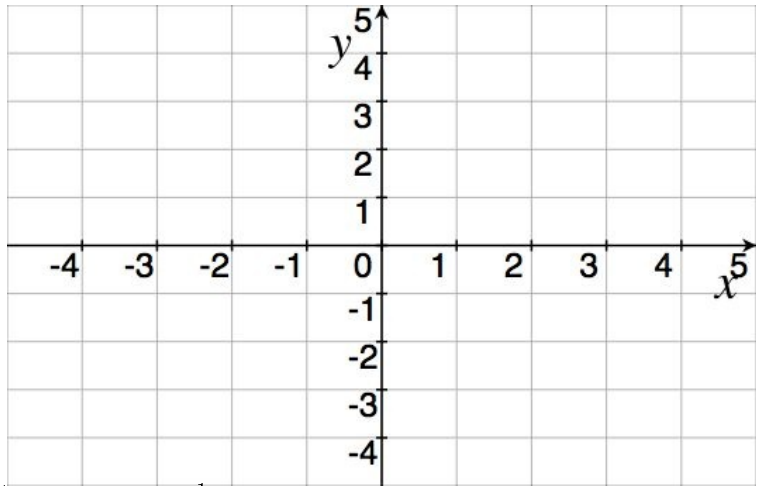
Graphs of Functions & Their Inverses

1. Let m be the function completely defined by the table:

$\star$	$m(\star)$	$\star$	$m^{-1}(\star)$
1	-3	-3	
$\frac{3}{2}$	2	2	
π	$\sqrt{2}$	$\sqrt{2}$	



   (a) Complete the table above to define m^{-1}.

   (b) Plot the graph of m on the set of axes provided.

   (c) Use a different mark (or color) to graph m^{-1} on the same set of axes.

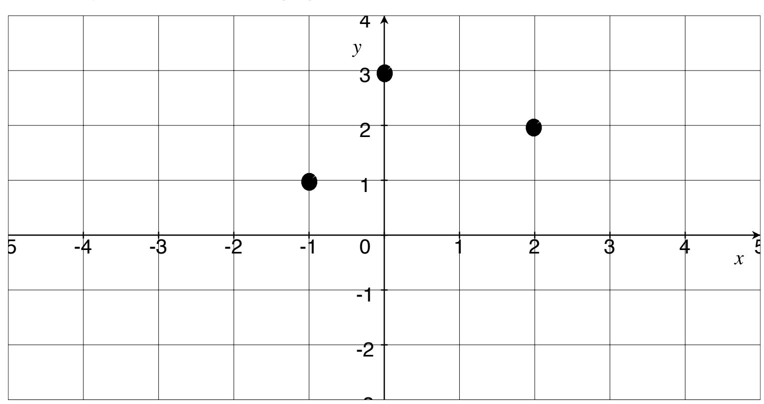
   (d) Notice the point (1,-3) is on the graph of m and (-3,1) is on the graph of m^{-1}. Similarly $(\frac{3}{2}, 2)$ is on the graph of m and $(2, \frac{3}{2})$ is on the graph of m^{-1}.

   (e) Find the domain of m and range of m^{-1}. Are there any similarities?

   (f) Find the range of m and the domain of m^{-1}. Are there any similarities?

   The observations you made in (e) & (f) are true in general, that is:
   if f is the inverse of g then: Domain of f=Range of g Range of f=Domain of g

2. Let n be the function defined by the following graph:



   (a) Will n have an inverse? Why?

   (b) Use the observations you made from #1 to graph n^{-1}.

   (c) Let d be the function defined by $d(x) = x$. Given that the graph of d is a straight line, plot some points of d and draw its graph.

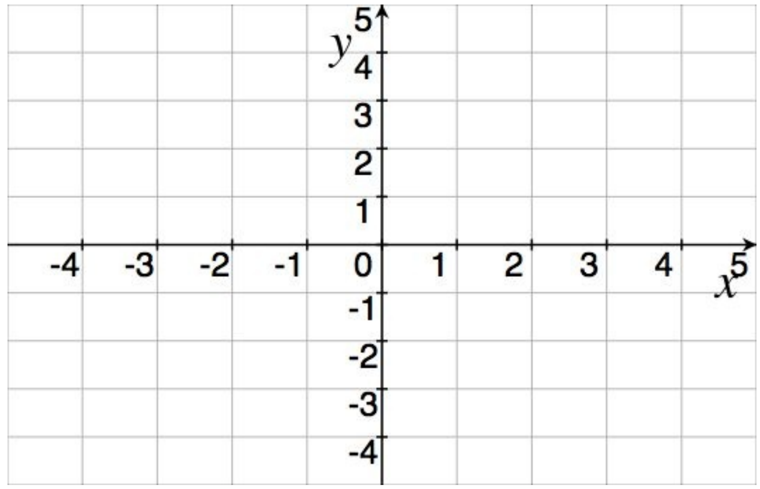
   (d) Do you see any symmetry between the graphs of n, n^{-1}, and d?

Verify your answers to #2d by looking at the box on the bottom of page 97.

3. Let p be the function defined by $p(x) = x^2 - 1$.

   (a) Draw the graph of p.

   (b) Will p have an inverse? Why?



   (c) Let the function q have the same rule as p (so $q(x) = x^2 - 1$), but with a *restricted* domain. The domain of q is set to all $x \geq 0$ (in interval notation: $[0, \infty)$). Draw the graph of q with distinct marks from the graph of p.

   (d) Will q have an inverse? Why

   (e) Identify a point (coordinates) on the graph of q and from this and the observations made in #1d, identify a point on the graph of q^{-1}.

   (f) Recall your observations made in #2d and try to sketch the graph of q^{-1} on the above set of axes. Verify your answer by looking at example 8 on page 101.

   (g) What is the domain and range of q^{-1}?

   When we are given a function that is *not* one-to-one we can choose to restrict the domain to a subsection and in so doing, define a partial inverse.

   (h) Could you have made a different partial inverse for the function p? How?