Practice TQS 120 Final

NAME: This is a sample final to be used for practice. This is *not a template* for the Final that will be given in class. Many of the questions on the Final will look quite di?erent than those appearing here.

[10] Let f & g, be functions.

T F $(f \circ g)(x) = (g \circ f)(x)$ ex $f(x) = x+1$ $g(x) = 2x$

$(f \circ g)(x) = 2x+1$ but $(g \circ f)(x) = 2(x+1)$

T F $(\frac{f}{g})(x) = (\frac{g}{f})(x)$

T F $\sqrt{(x^2)} = x$ for all real numbers x. let x be -1

T F If $h(x) = x^2 + 1$, then h is an even function.

sym w.r.t. y-axis

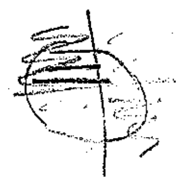
T F $\ln \frac{x}{y} = \ln x - \ln y$ for all non-negative numbers x and y.

T F $\log(\log(10)) = 0$. $\log(1) = 0$

T F Just as every integer is either even or odd, every function is either an even function or odd function. HW problem ex $g(x) = (x+1)^3$

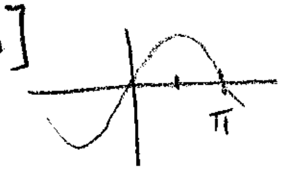
T F $\sin(\frac{\pi}{3} + x) = \sin \frac{\pi}{3} + x$

T F If $\sin\theta > 0$ and $\tan\theta < 0$, then $\cos\theta < 0$ in quadrant II



T F The range of $\sin^{-1}$ is $[0, \pi]$

$[-\pi/2, \pi/2]$

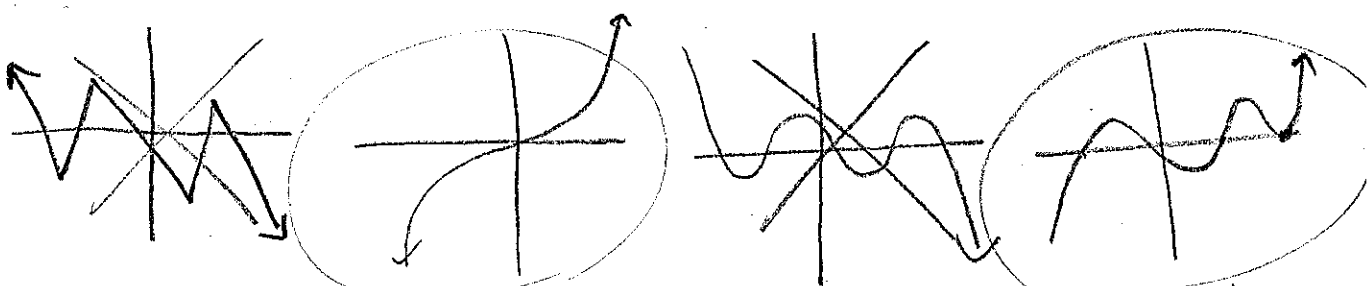


Right answers will *not* get credit without supporting work. Note "undefined" and "no solution" are possible answers.

1. [2] Explain what a function is.

A function is a domain (set of inputs), a range (set of outputs) and a rule between them assigning an input to exactly one output.

2. [2] Which of the following may be a graph of a polynomial of degree five with a positive leading coefficient?



3. You are conducting two experiments and are tracking the amount of oxygen. ~~You will~~ be taking your measurements in moles which is a common unit of measurement in chemistry that is the same as 6.0221415×10^{23} atoms.

You find if you start with x moles of oxygen, Experiment A returns $\frac{3}{x} - 4$ moles of oxygen.

However, if you start with x moles of oxygen, Experiment B returns $2\ln(x) - 1$ moles of oxygen.

(a) Let f_A and f_B be the functions that return the amount of moles of oxygen after Experiment A and B respectively. Write down the rule of f_A and f_B.

$$f_A(x) = 3/x - 4$$
$$f_B(x) = 2\ln(x) - 1$$

(b) Find a formula that returns the number of moles of oxygen if you start with x moles and run Experiment A and then Experiment B.

$$(f_B \circ f_A)(x) = f_B(3/x - 4) = 2\ln(3/x - 4) - 1$$

(c) What is the smallest amount of oxygen that can be put into Experiment A and then into Experiment B and still return the number predicted by your rule in (b)?

Domain question need $3/x - 4 > 0$ b/c domain of ln

$\Rightarrow 3/x > 4 \Rightarrow \boxed{\text{if } x > 0 \text{ then } 3/4 > x}$

if $x <$ ~~0 then~~ $3/4 < x$

b/c of the domain of ln

(d) If you ran Experiment B backwards and started with y moles of oxygen, how many moles would you be left with?

$f_B(x) = y$

$2\ln x - 1 = y$

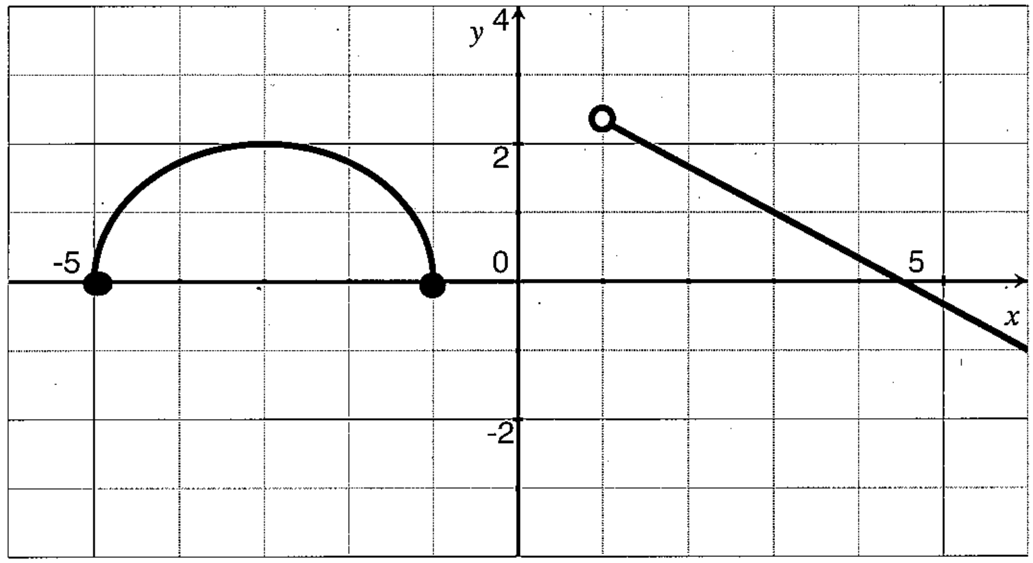
$2\ln x = y + 1$

$\ln x = \frac{y+1}{2}$

$x = e^{\frac{y+1}{2}}$

so $e^{\frac{y+1}{2}}$

4. [3] Let the following be the graph of g.



(a) What is the domain of g?

$[-5,-1] \cup (1,\infty)$

(b) The function g is a piecewise defined function consisting of a straight line and a semicircle. Write down the rule for g.

circle $\Rightarrow$ $(x+3)^2+y^2=2^2$

$y^2 = 4-(x+3)^2$

b/c upper half $y=\sqrt{4-(x+3)^2}$

line $\Rightarrow$ $\frac{-2}{3}$ is slope

thru $(3,1)$ $\Rightarrow 1 = -\frac{2}{3}(3)+b$

$\Rightarrow 1+2=b$

$y = -\frac{2}{3}x+3$

$$f(x) = \begin{cases} \sqrt{4-(x+3)^2} & \text{if } -5 \le x \\ -\frac{2}{3}x+3 & \text{if } x>1 \end{cases}$$

(c) Find the exact x value(s) so that $g(x)=2$?

when

$\sqrt{4-(x+3)^2}=2$

$4-(x+3)^2=4$

$(x+3)^2=0$

$\Rightarrow x=-3$

and

$-\frac{2}{3}x+3=2$

$-\frac{2}{3}x=-1$

$x=+\frac{3}{2}$

(d) Find the equation for a line that is perpendicular to the line with endpoints $(3,1)$ and $(6,-1)$. (There are many right answers.)

slope of line connecting $(3,1)$ & $(6,-1)$ is $-\frac{2}{3}$

so slope of $\perp$ line is $\frac{3}{2}$

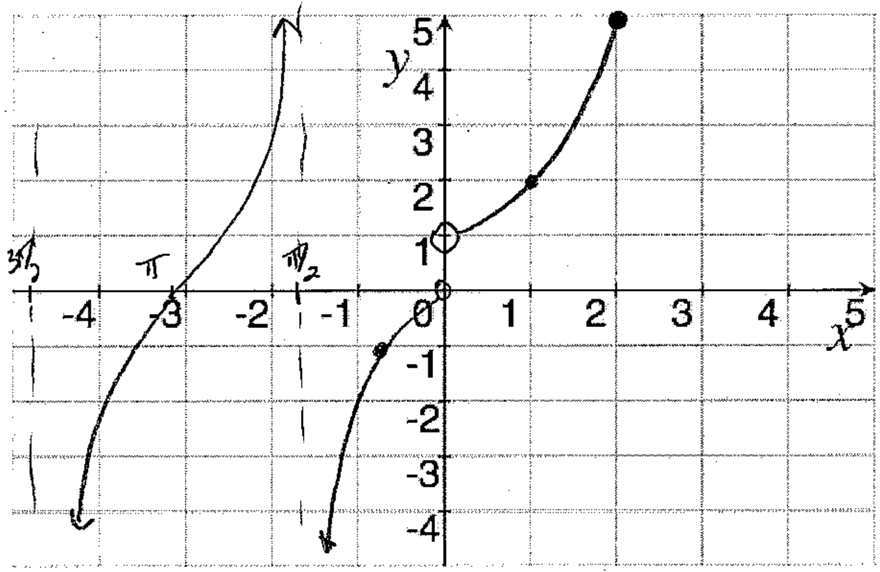
so $y=\frac{3}{2}x$ works.

5. Define f by

$$f(x) = \begin{cases} \tan x & \text{if } x < 0 \\ x^2 + 1 & \text{if } 0 < x \leq 2 \\ 2^{x+1} & \text{if } x \geq 2 \end{cases}$$

parabola shifted up 1
exp shifted left one

off the chart probably should have chosen 2^{x-1}



(a) [8] Graph f on the axes above.

(b) [9] Find the following if possible:

$f(1)$

$(1)^2 + 1 = 2$

$f(2) + f(3)$

$(2)^2 + 1 + 2^{3+1}$

$5 + 2^4 = 5 + 16 = 21$

$f(0)$

not defined

$f(\frac{-13\pi}{4})$

$\tan\left(\frac{-13\pi}{4}\right) = \tan\left(-\frac{12\pi}{4} - \frac{\pi}{4}\right)$

$= \tan(-3\pi - \pi/4)$ b/c tan has period

$= \tan(-\pi/4) = -1$

Range of f $\mathbb{R}$

by the way:
the domain is:
every real # but
$0, -\frac{\pi}{2}, -\frac{3\pi}{2}, -\frac{5\pi}{2}, -\frac{7\pi}{2}, \ldots$

6. [3] If $f(x)$ is an even function, $f(2) = 6$, and $g(x) = \frac{1}{2}f(2x) - \frac{1}{3}$, what is $g(-1)$?

$g(-1) = \frac{1}{2} f(2(-1)) - 1/3 = 1/2 f(-2) - 1/3$

since f is even, $f(-2) = f(2)$ so

$g(-1) = 1/2 f(2) - 1/3 = 1/2 \cdot 6 - 1/3 = 3 - 1/3 = 8/3$

7. [6] Assuming that $\log_3 x = 5.3$ and $\log_3 y = 2.1$ find the following exactly:

$\log_3 \frac{27x^3}{y^2}$

$= \log_3 27x^3 - \log_3 y^2$

$= \log_3 27 + \log_3 x^3 - 2\log_3 y$

$= \log_3 3^3 + 3\log_3 x - 2\log_3 y$

$= 3 + 3 \cdot 5.3 - 2 \cdot 2.1 = 14.7$

$\log_9 3x$

note $\log_3 x = 5.3$

$\Rightarrow x = 3^{5.3}$

so

$\log_9 3x = \log_9 3 \cdot 3^{5.1} = \log_9 3^{1+5.1}$

$= \log_9 3^{6.1} = \log_9 (9^{\frac{1}{2}})^{6.1} = \log_9 9^{3.}$

$= 3.05$

8. [4] Find all exact values for x that satisfy the following:

$\log(x - 16) = 2 - \log(x - 1)$

$\log(x-16) + \log(x-1) = 2$

$10^{\log (x-16)(x-1)} = 10^2$

$(x-16)(x-1) = 100$

$x^2 - 17x + 16 = 100$

$x^2 - 17x - 84 = 0$

$\Rightarrow x = \frac{17 \pm \sqrt{17^2 - 4(-84)}}{2(1)}$

$= \frac{17 \pm \sqrt{289 + 336}}{2}$

$= \frac{17 \pm \sqrt{625}}{2}$

need to throw out the neg answer b/c of domains.

$3^{5x}9^x = 27$

$3^{5x}(3^2)^x = 3^3$

$3^{5x} 3^{2x} = 3^3$

$3^{7x} = 3^3$

$\cancel{\log_3} 3^{7x} = \cancel{\log_3} 3^3$

$7x = 3$

$x = 3/7$

$$\begin{array}{r} 17 \\ 17 \\ \hline 119 \\ 170 \\ \hline 289 \end{array}$$

9. Simplify:

$\frac{\sqrt{c^2d^6}}{\sqrt{4c^3d^{-4}}}$

$\frac{(c^2d^6)^{1/2}}{(4c^3d^{-4})^{1/2}} = \frac{(c^2)^{1/2}(d^6)^{1/2}}{4^{1/2}(c^3)^{1/2}(d^{-4})^{1/2}}$

$= \frac{c d^3}{2c^{3/2}d^{-2}} = \frac{c^{1-3/2} d^{3-2}}{2}$

$= \frac{c^{-1/2} d^5}{2} = \frac{d^5}{2c^{1/2}} = \frac{d^5}{2\sqrt{c}}$

$\log_2 \frac{1}{4}$

$= \log_2 \frac{1}{2^2}$

$= \log_2 2^{-2}$

$= -2$

10. Simplify:

$$\frac{(x^2)^{\frac{1}{3}}(8y^2)^{\frac{2}{3}}}{4x^{\frac{2}{3}}y^2} = \frac{\cancel{x^{2/3}}\, 8^{2/3}(y^2)^{2/3}}{4\cancel{x^{2/3}}\, y^2}$$

$$= \frac{2y^{4/3}}{4y^2} = \frac{y^{4/3-2}}{2} = \frac{y^{-2/3}}{2} = \frac{1}{2y^{2/3}}$$

$2 - \log_5(25z)$

$\log_5 5^2 - \log_5 25z$

$\log_5 25 - \log_5 25z$

$\log_5 \frac{25}{25z} = \log_5 \frac{1}{z}$

$\sin^{-1}(\sin \frac{3\pi}{4})$

$\sin^{-1}(\frac{1}{\sqrt{2}}) = ?$

$\Rightarrow \sin ? = 1/\sqrt{2}$

$\Rightarrow ? = \pi/4$

$$\frac{\cos x}{1 - \sin x} + \frac{1 - \sin x}{\cos x}$$

$$\frac{\cos^2 x + (1-\sin x)^2}{\cos x(1-\sin x)}$$

$$\frac{\cos^2 x + 1 - 2\sin x + \sin^2 x}{\cos x(1-\sin x)}$$

$$= \frac{\boxed{\cos^2 x + \sin^2 x} + 1 - 2\sin x}{\cos x(1-\sin x)} = \frac{1+1-2\sin x}{\cos x(1-\sin x)}$$

$$= \frac{2-2\sin x}{\cos x(1-\sin x)} = \frac{2\cancel{(1-\sin x)}}{\cos x\cancel{(1-\sin x)}} = \frac{2}{\cos x}$$

11. [7] Given $f(3) = 0$, use the factor theorem to find the other roots of $x^4 - 3x^3 - 25x^2 + 75x$ $= f(x)$

b/c $f(3) = 0$

$(x-3)$ is a factor of f

$$\begin{array}{r|l} & x^3 - 25x \\ \hline x-3 & x^4 - 3x^3 - 25x^2 + 75x \\ & -(x^4 - 3x^3) \\ \hline & 0 - 25x^2 + 75x \\ & (-25x^2 + 75x) \\ \hline & 0 \end{array}$$

$\Rightarrow f(x) = x^4 - 3x^3 - 25x^2 + 75x$

$= (x-3)(x^3 - 25x)$

$= (x-3)\, x\, (x^2 - 25)$

$= (x-3)\, x\, (x+5)(x-5)$

the other roots are

$3, 0, -5 \,\&\, 5.$

12. [5] Your given a 16 oz mocha that is a rather weak 3% espresso. You, knowing you'll be up late studying mathematics, would rather like a 30% espresso drink. Realizing this you purchase an espresso machine. How much weak mocha do you discard and replace with straight espresso to have a 16 oz mocha with the desired concentration?

let x be the mocha you keep

y be the amount of espresso

$$\begin{array}{r} 16 \\ \times 3 \\ \hline 48 \end{array}$$

total mocha $\Rightarrow 16 = x + y$

total espresso $\Rightarrow .3 \cdot 16 = .03x + y$

2 equations, 2 unknowns

$16 = x + y$ and $4.8 = .03x + y$

$\Rightarrow 16 - y = x$ sub into

to get $4.8 = .03(16 - y) + y$

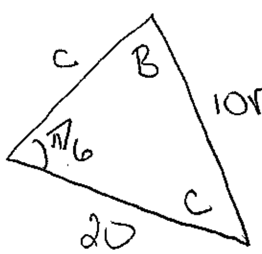
$4.8 = .48 - .03y + y$

$= .48 - .48$

$4.32 = .97y$

$\Rightarrow y = \frac{4.32}{.97} = \frac{432}{97}$ oz

13. [5] Use the conventions from the book and class and let A be measure of the angle opposite the side with length a. Given that $a = 10\sqrt{2}$, $b = 20$, and $A = \frac{\pi}{6}$ with the standard notation, determine if the information describes 0, 1, or 2 triangles and solve for them/it if they/it exist/s.



$$\frac{\sin \pi/6}{10\sqrt{2}} = \frac{\sin B}{20}$$

$$\frac{20 \sin \pi/6}{10\sqrt{2}} = \sin B$$

$$\frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} \cdot \frac{1}{2} = \sin B$$

$\Rightarrow B = \pi/4$ or $3\pi/4$

Recall

$\sin(\theta + \varphi) = \sin\theta\cos\varphi + \cos\theta\sin\varphi$

if $B = \pi/4$

$C = \pi - \pi/4 - \pi/6$

$= \frac{12\pi - 3\pi - 2\pi}{12}$

$\therefore \boxed{C = 7\pi/12}$

$$\Rightarrow \frac{\sin \pi/6}{10\sqrt{2}} = \frac{\sin \frac{7\pi}{12}}{c}$$

$$\frac{1}{20\sqrt{2}} = \frac{\sin \frac{7\pi}{12}}{c}$$

$c = 20\sqrt{2} \sin \frac{7\pi}{12}$

$c = 20\sqrt{2} \sin\left(\frac{3\pi}{12} + \frac{4\pi}{12}\right)$

$c = 20\sqrt{2}\left[\sin \frac{\pi}{4} \cos \frac{\pi}{3} + \cos \frac{\pi}{4} \sin \frac{\pi}{3}\right]$

$\boxed{c = 20\sqrt{2}\left[\frac{1}{\sqrt{2}}\frac{1}{2} + \frac{1}{\sqrt{2}}\frac{\sqrt{3}}{2}\right] = 10(1+\sqrt{3})}$

if $B = 3\pi/4$

$C = \pi - 3\pi/4 - \pi/6$

$= \frac{12\pi - 9\pi - 2\pi}{12}$

$\therefore \boxed{C = \pi/12}$

$$\frac{\sin \pi/6}{10\sqrt{2}} = \frac{\sin \pi/12}{c}$$

$$\frac{1}{20\sqrt{2}} = \frac{\sin \pi/12}{c}$$

$c = 20\sqrt{2} \sin \pi/12$

$c = 20\sqrt{2} \sin\left(\frac{4\pi}{12} - \frac{3\pi}{12}\right)$

$c = 20\sqrt{2}\left[\sin \frac{\pi}{3} \cos \frac{\pi}{4} + \cos \frac{\pi}{3} \sin\right.$

$\boxed{c = 10(\sqrt{3} - 1)}$

14. [5] Suppose a radioactive isotope is such that one-fifth of the atoms in a sample decay after three years. Find the half-life of this isotope

use $P_0 2^{-t/h} = P(t)$

start with P_0 & end with $\frac{4}{5}P_0$ when $t=3$.

$$\frac{\frac{4}{5}\cancel{P_0}}{\cancel{P_0}} = \frac{\cancel{P_0}\, 2^{-3/h}}{\cancel{P_0}}$$ solve for h.

$$\frac{4}{5} = 2^{-3/h}$$

$$\ln \frac{4}{5} = \frac{-3}{h}\ln 2$$

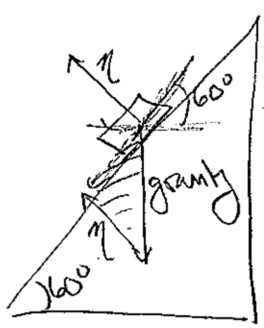
$$\frac{\ln \frac{4}{5}}{\ln 2} = \frac{-3}{h}$$

$$\Rightarrow h\,\frac{\ln \frac{4}{5}}{\ln 2} = -3$$

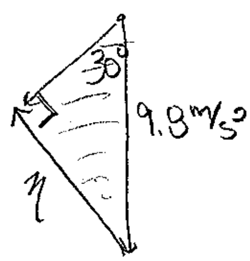
$$\Rightarrow h = \frac{-3\ln 2}{\ln \frac{4}{5}}$$

15. [5] The force of friction is sometimes calculated by multiplying the normal force (the force holding the object up) by the mass of the object and by a 'coefficient of friction'. The coefficient of friction is a dimensionless number that depends on the two surfaces being pressed together.

A 10kg block is sliding down a dry glass ramp with angle of elevation of 60° and with a coefficient of friction of .94. Find the force of friction acting on the block.



need to find η & then compute $.94\eta\cdot$(mass of obje
ie $.94\eta\cdot 10$kg



Sohcahtoa

$$\sin 30^\circ = \frac{\eta}{9.8}$$

$$\Rightarrow \eta = 9.8\sin 30^\circ = 9.8\cdot\tfrac{1}{2}$$

So the force of friction is $.94\cdot 9.8\cdot .5\cdot 10\ \text{m/s}^2\cdot\text{kg}$