Practice TMath 120

NAME: This is a sample final to be used for practice. This is *not a template* for the Final that will be given in class. Many of the questions on the Final will look quite di?erent than those appearing here.

Final

[10] Let f & g, be functions.

- T F $(f \circ g)(x) = (g \circ f)(x)$
- T F $\left(\frac{f}{g}\right)(x) = \left(\frac{g}{f}\right)(x)$
- T F $\sqrt{(x^2)} = x$ for all real numbers x.
- T F If 2 is a root of g, then g(2) = 0.
- T F $\ln \frac{x}{y} = \ln x \ln y$ for all positive numbers x and y.
- $T \quad F \quad \log(\log(10)) = 0.$
- T F $\sin^{-1}(\sin x) = x$ for all real numbers x.

T F
$$\sin(\frac{\pi}{3} + x) = \sin\frac{\pi}{3} + x$$

- T F If $\sin \theta > 0$ and $\tan \theta < 0$, then $\cos \theta < 0$
- T F The range of \sin^{-1} is $[0, \pi]$

Right answers will *not* get credit without supporting work. Note "undefined" and "no solution" are possible answers.

1. Find all x such that

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$$2(5 - (8 - x)^2)^{-\frac{1}{2}} - 1 = 0$$

2. [2] Explain what a function is.

- 3. Given $m(x) = \frac{x}{x-5}$, and $n(x) = \sqrt{4x-8}$,
 - (a) The inverse to the function m exists. Find m^{-1} .

(b) [4] If p(x) = 3m(x+1), find the domain and rule of p.

(c) [3] Find the domain and rule of $n \circ m$.

(d) [5] Find the domain and rule of $\frac{n}{m}$.

4. [3] Let the following be the graph of g.



- (a) What is the domain of g?
- (b) The function g is a piecewise defined function consisting of a straight line and a semicircle. Write down the rule for g.

- (c) Use the graph above to *estimate* all x value(s) so that g(x) = 1?
- (d) Find the total length (of the curve and the line) that is graphed above.

5. Define f by



- (a) [8] Graph f on the axes above.
- (b) [9] Find the following if possible: f(1) f(2) + f(3)

$$f(\frac{-13\pi}{4})$$

Range of
$$f$$

f(0)

6. [8] Find all of the exact values x that satisfy the following: $2\sin x = -\sqrt{3} \qquad \qquad 5^{4x-1} = 7^x$

7. [4] Find all exact values for x that satisfy the following: $\log(x - 16) = 2 - \log(x - 1)$ $3^{5x}9^x = 27$

8. Assume c, d, and z are all greater than zero and simplify:

$$\frac{\sqrt{c^2 d^6}}{\sqrt{4c^3 d^{-4}}} \qquad \qquad 2 - \log_5(25z)$$

9. [7] Given f(3) = 0, use the connection between roots and factors to find the other roots of $f(x) = x^4 - 3x^3 - 25x^2 + 75x$

10. Simplify:

$$\sin^{-1}(\sin\frac{3\pi}{4}) \qquad \qquad \frac{\cos x}{1-\sin x} + \frac{1-\sin x}{\cos x}$$

11. [4] Let $\frac{-\pi}{2} < \theta < 0$ and $\cos \theta = \frac{1}{5}$. Find $\tan \theta$.

12. [6] Let $\frac{\pi}{2} < \phi < \pi$ and $\frac{-\pi}{2} < \theta < 0$. Given that $\sin \phi = \frac{2}{3}$ and that $\cos \theta = \frac{1}{5}$, find $\cos(\theta + \phi)$ exactly. (You are free to use results from #10 above.)

- 13. Provide a graph AND an algebraic rule for each of the functions described below:
 - (a) A sixth degree polynomial with 6 distinct roots.



(b) A second degree polynomial with no real roots.



(c) A rational function with a vertical asymptote at x=2.



(d) A trigonometric function with period $\frac{\pi}{2}$.



(e) A piece-wise defined graph with a jump in the graph when x = 3.

				v ⁵					
				4					
				3					
				2					
				1					
-4	-3	-2	-1	0	1	2	3	4	,5
				-1					A
				-2					
				-3					
		-		-4				-	

(f) A logarithmic graph with $y \to -\infty$ when $x \to -3$.



14. [5] Your given a 16 oz mocha that is a rather weak 3% espresso. You, knowing you'll be up late studying mathematics, would rather like a 30% espresso drink. Realizing this you purchase an espresso machine. How much weak mocha do you discard and replace with straight espresso to have a 16 oz mocha with the desired concentration?

15. [5] Use the conventions from the book and class and let A be measure of the angle opposite the side with length a. Given that $a = 10\sqrt{2}$, b = 20, and $A = \frac{\pi}{6}$ with the standard notation, determine if the information describes 0, 1, or 2 triangles and solve for them/it if they/it exist/s.

16. Suppose a radioactive isotope is such that one-fifth of the atoms in a sample decay after three years. Find the half-life of this isotope

- 17. Points A and B are separated by a lake. To find the distance between them, a surveyor locates a point on land such that $\angle CAB = 30^{\circ}$. She also measures CA as 475ft and CB as 345ft.
 - (a) [2] Draw a picture of the situation. Do you have enough information to find the distance between A and B? Justify yourself.
 - (b) [3] If you can, find the distance between A and B. If you can't, what two distances are possible?