NAME: This is a sample final to be used for practice. This is not a template for the Final that will be given in class. Many of the questions on the Final will look quite di?erent than those appearing here.
[10] Let $f \& g$, be functions.
$\mathrm{T} \quad \mathrm{F} \quad(f \circ g)(x)=(g \circ f)(x)$
$\mathrm{T} \quad \mathrm{F} \quad\left(\frac{f}{g}\right)(x)=\left(\frac{g}{f}\right)(x)$
$\mathrm{T} \quad \mathrm{F} \quad \sqrt{\left(x^{2}\right)}=x$ for all real numbers $x$.

T F If 2 is a root of $g$, then $g(2)=0$.
T F $\quad \ln \frac{x}{y}=\ln x-\ln y$ for all positive numbers $x$ and $y$.
T $\quad \mathrm{F} \quad \log (\log (10))=0$.
$\mathrm{T} \quad \mathrm{F} \quad \sin ^{-1}(\sin x)=x$ for all real numbers $x$.
T $\quad \mathrm{F} \quad \sin \left(\frac{\pi}{3}+x\right)=\sin \frac{\pi}{3}+x$
T F If $\sin \theta>0$ and $\tan \theta<0$, then $\cos \theta<0$
$\mathrm{T} \quad \mathrm{F} \quad$ The range of $\sin ^{-1}$ is $[0, \pi]$
Right answers will not get credit without supporting work. Note "undefined" and "no solution" are possible answers.

1. Find all $x$ such that

$$
2\left(5-(8-x)^{2}\right)^{-\frac{1}{2}}-1=0
$$

2. [2] Explain what a function is.
3. Given $m(x)=\frac{x}{x-5}$, and $n(x)=\sqrt{4 x-8}$,
(a) The inverse to the function $m$ exists. Find $m^{-1}$.
(b) [4] If $p(x)=3 m(x+1)$, find the domain and rule of $p$.
(c) [3] Find the domain and rule of $n \circ m$.
(d) [5] Find the domain and rule of $\frac{n}{m}$.
4. [3] Let the following be the graph of $g$.

(a) What is the domain of $g$ ?
(b) The function $g$ is a piecewise defined function consisting of a straight line and a semicircle. Write down the rule for $g$.
(c) Use the graph above to estimate all $x$ value(s) so that $g(x)=1$ ?
(d) Find the total length (of the curve and the line) that is graphed above.
5. Define $f$ by

$$
f(x)= \begin{cases}\tan x & \text { if } x<0 \\ x^{2}+1 & \text { if } 0<x \leq 2 \\ \log _{2}(x-1) & \text { if } 2<x\end{cases}
$$

|  |  |  |  | $y_{4}^{5}$ |  |  |  |  |  |
| :--- | :--- | :--- | :--- | ---: | :--- | :--- | :--- | :--- | :--- |

(a) [8] Graph $f$ on the axes above.
(b) [9] Find the following if possible: $f(1)$

$$
f(2)+f(3)
$$

$f(0)$

$$
f\left(\frac{-13 \pi}{4}\right)
$$

Range of $f$
6. [8] Find all of the exact values $x$ that satisfy the following:

$$
2 \sin x=-\sqrt{3}
$$

$$
5^{4 x-1}=7^{x}
$$

7. [4] Find all exact values for $x$ that satisfy the following:

$$
\log (x-16)=2-\log (x-1) \quad 3^{5 x} 9^{x}=27
$$

8. Assume $c, d$, and $z$ are all greater than zero and simplify:

$$
\frac{\sqrt{c^{2} d^{6}}}{\sqrt{4 c^{3} d^{-4}}}
$$

$$
2-\log _{5}(25 z)
$$

9. [7] Given $f(3)=0$, use the connection between roots and factors to find the other roots of $f(x)=x^{4}-3 x^{3}-25 x^{2}+75 x$
10. Simplify:

$$
\sin ^{-1}\left(\sin \frac{3 \pi}{4}\right)
$$

$$
\frac{\cos x}{1-\sin x}+\frac{1-\sin x}{\cos x}
$$

11. [4] Let $\frac{-\pi}{2}<\theta<0$ and $\cos \theta=\frac{1}{5}$. Find $\tan \theta$.
12. [6] Let $\frac{\pi}{2}<\phi<\pi$ and $\frac{-\pi}{2}<\theta<0$. Given that $\sin \phi=\frac{2}{3}$ and that $\cos \theta=\frac{1}{5}$, find $\cos (\theta+\phi)$ exactly. (You are free to use results from \#10 above.)
13. Provide a graph AND an algebraic rule for each of the functions described below:
(a) A sixth degree polynomial with 6 distinct roots.

(b) A second degree polynomial with no real roots.

(c) A rational function with a vertical asymptote at $\mathrm{x}=2$.

(d) A trigonometric function with period $\frac{\pi}{2}$.

(e) A piece-wise defined graph with a jump in the graph when $x=3$.

(f) A logarithmic graph with $y \rightarrow-\infty$ when $x \rightarrow-3$.

14. [5] Your given a 16 oz mocha that is a rather weak $3 \%$ espresso. You, knowing you'll be up late studying mathematics, would rather like a $30 \%$ espresso drink. Realizing this you purchase an espresso machine. How much weak mocha do you discard and replace with straight espresso to have a 16 oz mocha with the desired concentration?
15. [5] Use the conventions from the book and class and let $A$ be measure of the angle opposite the side with length $a$. Given that $a=10 \sqrt{2}, b=20$, and $A=\frac{\pi}{6}$ with the standard notation, determine if the information describes 0,1 , or 2 triangles and solve for them/it if they/it exist/s.
16. Suppose a radioactive isotope is such that one-fifth of the atoms in a sample decay after three years. Find the half-life of this isotope
17. Points $A$ and $B$ are separated by a lake. To find the distance between them, a surveyor locates a point on land such that $\angle C A B=30^{\circ}$. She also measures $C A$ as 475 ft and $C B$ as 345 ft .
(a) [2] Draw a picture of the situation. Do you have enough information to find the distance between $A$ and $B$ ? Justify yourself.
(b) [3] If you can, find the distance between $A$ and $B$. If you can't, what two distances are possible?
