

NAME: This is a sample final to be used for practice. This is *not a template* for the Final that will be given in class. Many of the questions on the Final will look quite different than those appearing here.

[10] Let f & g , be functions.

T F $(f \circ g)(x) = (g \circ f)(x)$

T F $(\frac{f}{g})(x) = (\frac{g}{f})(x)$

T F $\sqrt{(x^2)} = x$ for all real numbers x .

T F If 2 is a root of g , then $g(2) = 0$.

T F $\ln \frac{x}{y} = \ln x - \ln y$ for all positive numbers x and y .

T F $\log(\log(10)) = 0$.

T F $\sin^{-1}(\sin x) = x$ for all real numbers x .

T F $\sin(\frac{\pi}{3} + x) = \sin \frac{\pi}{3} + x$

T F If $\sin \theta > 0$ and $\tan \theta < 0$, then $\cos \theta < 0$

T F The range of \sin^{-1} is $[0, \pi]$

Right answers will *not* get credit without supporting work. Note “undefined” and “no solution” are possible answers.

1. Find all x such that

$$2(5 - (8 - x)^2)^{-\frac{1}{2}} - 1 = 0$$

.

2. [2] Explain what a function is.

3. Given $m(x) = \frac{x}{x-5}$, and $n(x) = \sqrt{4x-8}$,

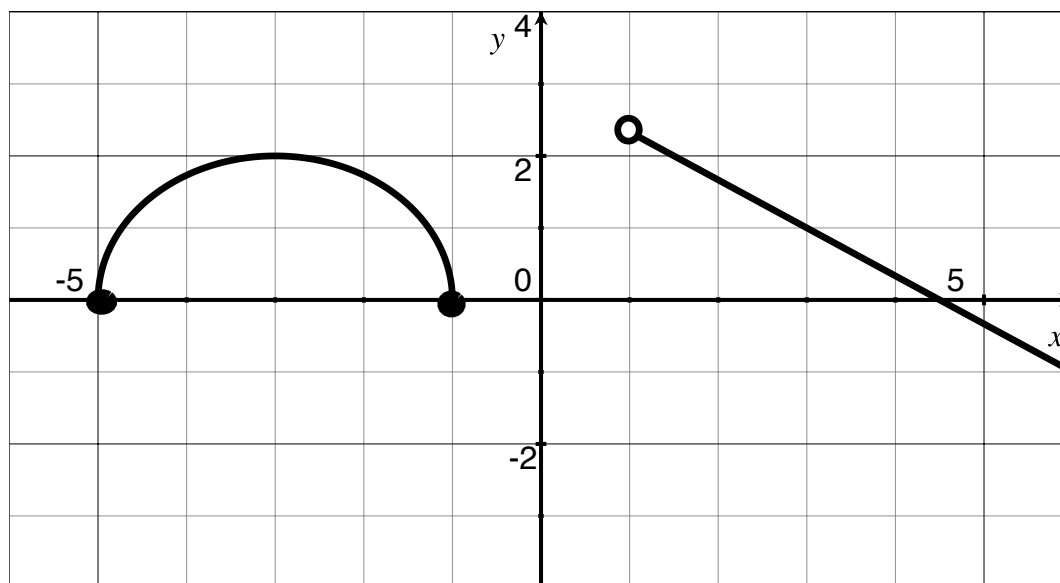
(a) The inverse to the function m exists. Find m^{-1} .

(b) [4] If $p(x) = 3m(x+1)$, find the domain and rule of p .

(c) [3] Find the domain and rule of $n \circ m$.

(d) [5] Find the domain and rule of $\frac{n}{m}$.

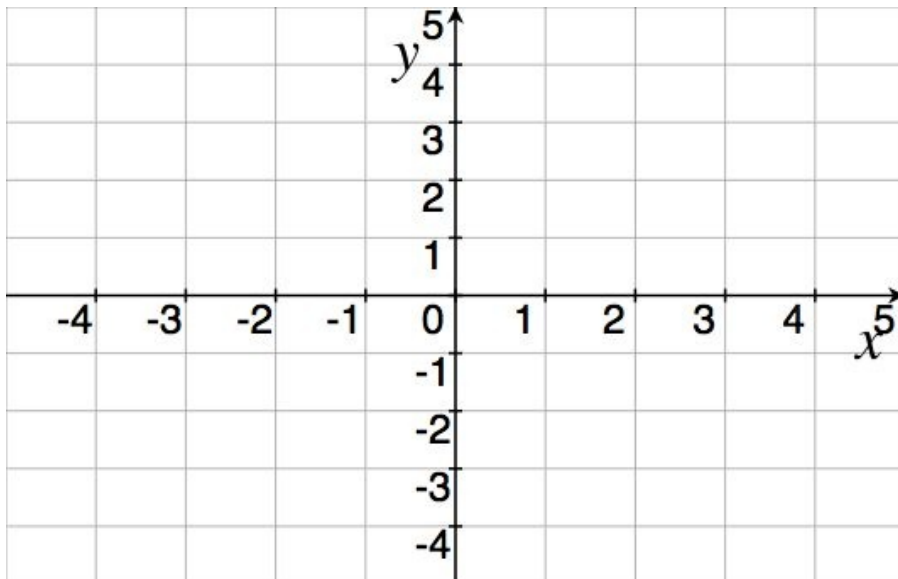
4. [3] Let the following be the graph of g .



- (a) What is the domain of g ?
- (b) The function g is a piecewise defined function consisting of a straight line and a semicircle. Write down the rule for g .
- (c) Use the graph above to *estimate* all x value(s) so that $g(x) = 1$?
- (d) Find the total length (of the curve and the line) that is graphed above.

5. Define f by

$$f(x) = \begin{cases} \tan x & \text{if } x < 0 \\ x^2 + 1 & \text{if } 0 < x \leq 2 \\ \log_2(x - 1) & \text{if } 2 < x \end{cases}$$



(a) [8] Graph f on the axes above.

(b) [9] Find the following if possible:

$$f(1)$$

$$f(2) + f(3)$$

$$f(0)$$

$$f\left(\frac{-13\pi}{4}\right)$$

Range of f

6. [8] Find all of the exact values x that satisfy the following:

$$2 \sin x = -\sqrt{3}$$

$$5^{4x-1} = 7^x$$

7. [4] Find all exact values for x that satisfy the following:

$$\log(x - 16) = 2 - \log(x - 1)$$

$$3^{5x} 9^x = 27$$

8. Assume c , d , and z are all greater than zero and simplify:

$$\frac{\sqrt{c^2 d^6}}{\sqrt{4c^3 d^{-4}}}$$

$$2 - \log_5(25z)$$

9. [7] Given $f(3) = 0$, use the connection between roots and factors to find the other roots of $f(x) = x^4 - 3x^3 - 25x^2 + 75x$

10. Simplify:

$$\sin^{-1}\left(\sin \frac{3\pi}{4}\right)$$

$$\frac{\cos x}{1 - \sin x} + \frac{1 - \sin x}{\cos x}$$

11. [4] Let $\frac{-\pi}{2} < \theta < 0$ and $\cos \theta = \frac{1}{5}$. Find $\tan \theta$.

12. [6] Let $\frac{\pi}{2} < \phi < \pi$ and $\frac{-\pi}{2} < \theta < 0$. Given that $\sin \phi = \frac{2}{3}$ and that $\cos \theta = \frac{1}{5}$, find $\cos(\theta + \phi)$ exactly. (You are free to use results from #10 above.)

13. [5] You are given a 16 oz mocha that is a rather weak 3% espresso. You, knowing you'll be up late studying mathematics, would rather like a 30% espresso drink. Realizing this you purchase an espresso machine. How much weak mocha do you discard and replace with straight espresso to have a 16 oz mocha with the desired concentration?
14. [5] Use the conventions from the book and class and let A be measure of the angle opposite the side with length a . Given that $a = 10\sqrt{2}$, $b = 20$, and $A = \frac{\pi}{6}$ with the standard notation, determine if the information describes 0, 1, or 2 triangles and solve for them/it if they/it exist/s.

15. Suppose a radioactive isotope is such that one-fifth of the atoms in a sample decay after three years. Find the half-life of this isotope
16. Points A and B are separated by a lake. To find the distance between them, a surveyor locates a point on land such that $\angle CAB = 30^\circ$. She also measures CA as 475ft and CB as 345ft.
- (a) [2] Draw a picture of the situation. Do you have enough information to find the distance between A and B ? Justify yourself.
 - (b) [3] If you can, find the distance between A and B . If you can't, what two distances are possible?