Practice TQS 120 Final

NAME: This is a sample final to be used for practice. This is *not a template* for the Final that will be given in class. Many of the questions on the Final will look quite di?erent than those appearing here.

[10] Let f & g, be functions.

T F $(f \circ g)(x) = (g \circ f)(x)$

T F $(\frac{f}{g})(x) = (\frac{g}{f})(x)$

T F $\sqrt{(x^2)} = x$ for all real numbers x.

T F If $h(x) = x^2 + 1$, then h is an even function.

T F $\ln \frac{x}{y} = \ln x - \ln y$ for all non-negative numbers x and y.

T F $\log(\log(10)) = 0$.

T F Just as every integer is either even or odd, every function is either an even function or odd function.

T F $\sin(\frac{\pi}{3} + x) = \sin \frac{\pi}{3} + x$

T F If $\sin\theta > 0$ and $\tan\theta < 0$, then $\cos\theta < 0$

T F The range of $\sin^{-1}$ is $[0, \pi]$

Right answers will *not* get credit without supporting work. Note "undefined" and "no solution" are possible answers.

1. [2] Explain what a function is.

2. [2] Which of the following may be a graph of a polynomial of degree five with a positive leading coefficient?

3. You are conducting two experiments and are tracking the amount of oxygen. You will be taking your measurements in moles which is a common unit of measurement in chemistry that is the same as 6.0221415×10^{23} atoms.

   You find if you start with x moles of oxygen, Experiment A returns $\frac{3}{x} - 4$ moles of oxygen.

   However, if you start with x moles of oxygen, Experiment B returns $2\ln(x) - 1$ moles of oxygen.

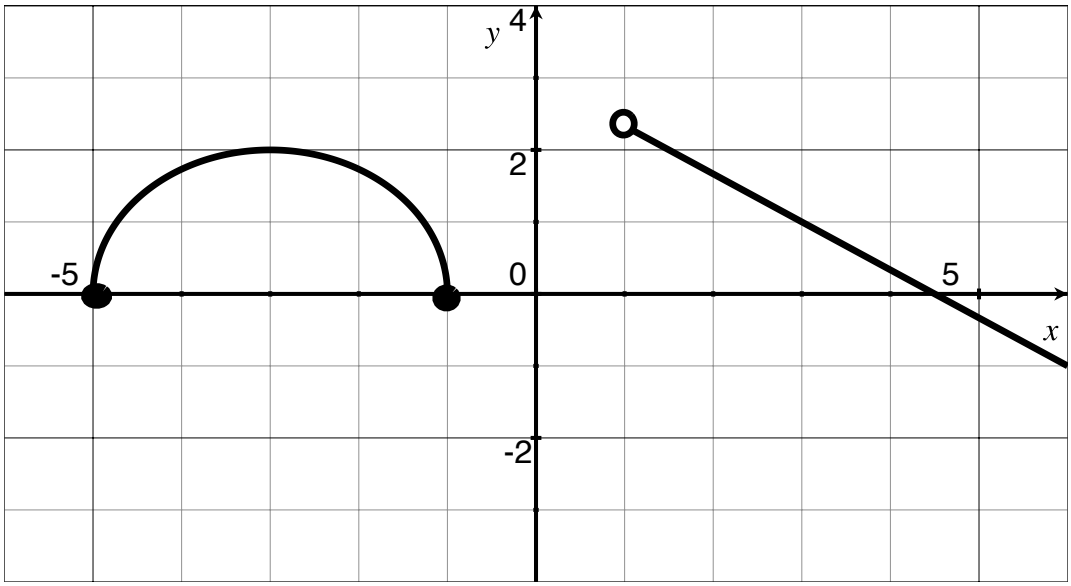
   (a) Let f_A and f_B be the functions that return the amount of moles of oxygen after Experiment A and B respectively. Write down the rule of f_A and f_B.

   (b) Find a formula that returns the number of moles of oxygen if you start with x moles and run Experiment A and then Experiment B.

   (c) What is the smallest amount of oxygen that can be put into Experiment A and then into Experiment B and still return the number predicted by your rule in (b)?

   (d) If you ran Experiment B backwards and started with y moles of oxygen, how many moles would you be left with?

4. [3] Let the following be the graph of g.



(a) What is the domain of g?

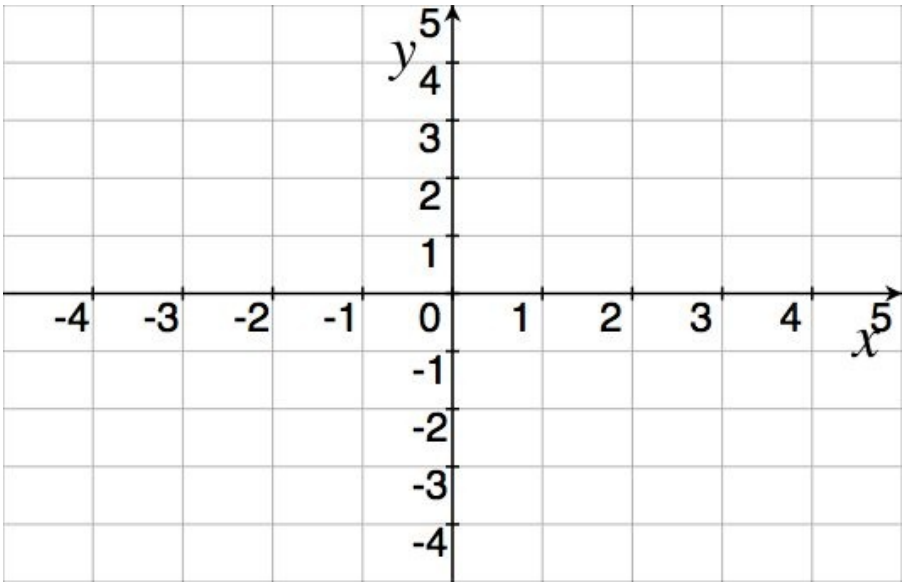
(b) The function g is a piecewise defined function consisting of a straight line and a semicircle. Write down the rule for g.

(c) Find the exact x value(s) so that $g(x) = 2$?

(d) Find the equation for a line that is perpendicular to the line with endpoints $(3, 1)$ and $(6, -1)$. (There are many right answers.)

5. Define f by

$$f(x) = \begin{cases} \tan x & \text{if } x < 0 \\ x^2 + 1 & \text{if } 0 < x \leq 2 \\ 2^{x+1} & \text{if } x \geq 2 \end{cases}$$



(a) [8] Graph f on the axes above.

(b) [9] Find the following if possible:

$f(1)$

$f(2) + f(3)$

$f(0)$

$f(\frac{-13\pi}{4})$

Range of f

6. [3] If $f(x)$ is an even function, $f(2) = 6$, and $g(x) = \frac{1}{2}f(2x) - \frac{1}{3}$, what is $g(-1)$?

7. [6] Assuming that $\log_3 x = 5.3$ and $\log_3 y = 2.1$ find the following exactly:

   $\log_3 \dfrac{27x^3}{y^2}$ $\qquad\qquad$ $\log_9 3x$

8. [4] Find all exact values for x that satisfy the following:

   $\log(x - 16) = 2 - \log(x - 1)$ $\qquad\qquad$ $3^{5x}9^x = 27$

9. Simplify:

   $\dfrac{\sqrt{c^2d^6}}{\sqrt{4c^3d^{-4}}}$ $\qquad\qquad$ $\log_2 \frac{1}{4}$

10. Simplify:

$\dfrac{(x^2)^{\frac{1}{3}}(8y^2)^{\frac{2}{3}}}{4x^{\frac{2}{3}}y^2}$

$2 - \log_5(25z)$

$\sin^{-1}(\sin\dfrac{3\pi}{4})$

$\dfrac{\cos x}{1-\sin x} + \dfrac{1-\sin x}{\cos x}$

11. [7] Given $f(3) = 0$, use the factor theorem to find the other roots of $x^4 - 3x^3 - 25x^2 + 75x$

12. [5] Your given a 16 oz mocha that is a rather weak 3% espresso. You, knowing you'll be up late studying mathematics, would rather like a 30% espresso drink. Realizing this you purchase an espresso machine. How much weak mocha do you discard and replace with straight espresso to have a 16 oz mocha with the desired concentration?

13. [5] Use the conventions from the book and class and let A be measure of the angle opposite the side with length a. Given that $a = 10\sqrt{2}$, $b = 20$, and $A = \frac{\pi}{6}$ with the standard notation, determine if the information describes 0, 1, or 2 triangles and solve for them/it if they/it exist/s.

14. [5] Suppose a radioactive isotope is such that one-fifth of the atoms in a sample decay after three years. Find the half-life of this isotope

15. [5] The force of friction is sometimes calculated by multiplying the normal force (the force holding the object up) by the mass of the object and by a 'coefficient of friction'. The coefficient of friction is a dimensionless number that depends on the two surfaces being pressed together.

    A 10kg block is sliding down a dry glass ramp with angle of elevation of 60° and with a coefficient of friction of .94. Find the force of friction acting on the block.