Exponent Properties

While working in a group make sure you:

- Expect to make mistakes but be sure to reflect/learn from them!
- Are civil and are aware of your impact on others.
- Assume and engage with the strongest argument while assuming best intent.

There are 'five' properties of exponents that you need to know well to work with algebraic expressions and equations. The 'zeroth' property is:

Property 0) Let b be a real number and m a positive integer, then

$$b^m = b \cdot b \cdot b \cdot \ldots \cdot b$$

where b appears m times on the right.

For example, we can write out $(-2)^4$ with no exponents as $(-2)\cdot(-2)\cdot(-2)\cdot(-2)$ or $(4)^3$ as $4 \cdot 4 \cdot 4$. We will often rely on the order of operations and drop parenthesis and just write 4^3 instead.

Warning: the order of operations is such that exponents *only* effect the object they 'hover' behind. For example,$-2^4 = -2 \cdot 2 \cdot 2 \cdot 2$, since the exponent is only effecting the 2 and not the negative sign.

There are some special cases of the above definition (that not all mathematicians agree on). Mainly:

$$0^m = 0 \qquad 1^m = 1 \qquad b^0 = 1 \qquad 0^0 \text{ is undefined}$$

1. Consider $x^2 \cdot x^5$.

   (a) Write out $x^2 \cdot x^5$ with no exponents (like what was done in the above paragraph).

   (b) Write $x^2 \cdot x^5$ as x raised to a single power.

2. Use your work above the finish the following first property:

Property 1) Let b be a real number, and let m and n be positive integers, then

$$b^m \cdot b^n =$$

1. Write out $\square^4$ with no exponents (like you did on the first page).

2. Write an $(x^3)^4$ using only multiplication and instances of x^3. In other words, do the same problem as number one but instead of using a square, use the symbol x^3.

3. Now expand the above with Property 0 to write out $(x^3)^4$ with no exponents.

4. Use your work above to finish the following:

Property 2) Let b be a real number, and let m and n be positive integers, then

$$(b^m)^n =$$

1. Now for some practice... simplify $\frac{-1}{5}^2(-5x^2y^3)^2(3x^3y)$.

1. Let x and y be real numbers and m be positive integers. Is there a relationship between $(x \cdot y)^m$ and $x^m \cdot y^m$? If so, state it and explain why it is true.

By answering the previous question you can now state:

Property 3) Let a and b be real numbers, and let m be a positive integer, then

$$(a \cdot b)^m =$$

You can verify these properties on page 14 of your text. I suggest you *do not* memorize them as most people mix them up. Rather, work through examples like 2 on page 1 and 4 on page 2 whenever you realize you need to work with exponents.

The remaining two properties are:

Property 4) Let b be a real number and let m and n be positive integers, then

$$\frac{b^m}{b^n} = b^{m-n}.$$

and

Property 5) Let a and b be a real numbers and let m be a positive integer, then

$$\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m.$$

1. Recall that $x^{-2} = \frac{1}{x^2}$. Make sure Property 4 is consistent with this observation by simplifying

$$\frac{x^6}{x^8}$$

All the rules can be found in Appendix 1 on pages 788 & 189. Now, some practice...

1. Simplify $\left(\dfrac{x^5}{2y^{-3}}\right)^{-3}$. Hint: use *lots* of room and make sure your exponents don't fall!

2. Solve for x (real or complex) given
   $3x^{-2} - 7 = 0$ $\qquad\qquad$ $(x+1)^{-1} + \frac{1}{2} = (x+3)^{-1}$

Notice that you can verify your answers to 1-2 by looking at Example 7 on page 790. Number 3 comes up a great deal in calculus word problems.