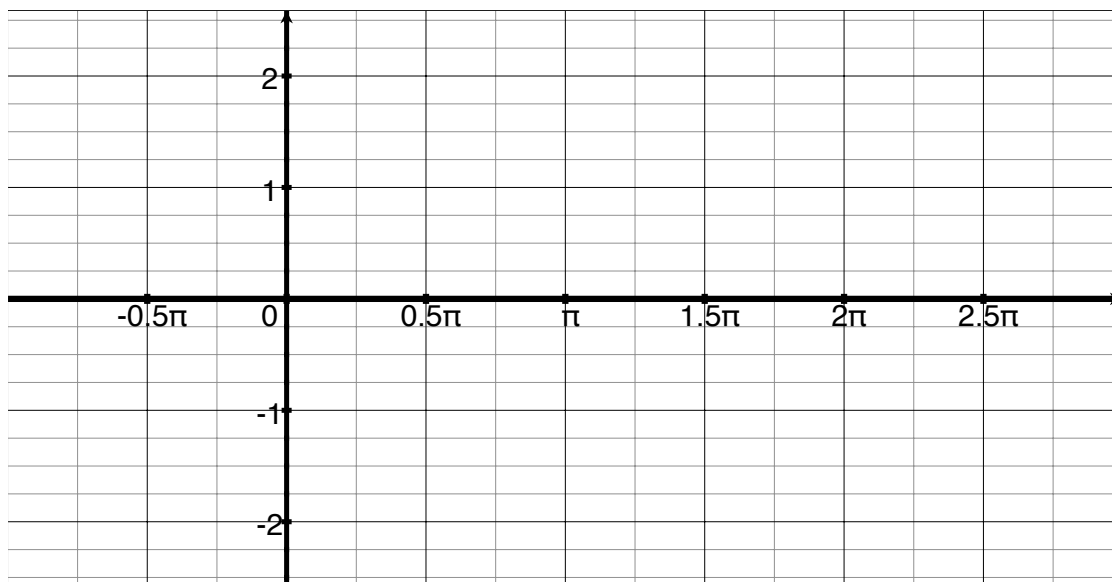


# Trigonometric Graph Transformations

1. Let  $f(\theta) = \cos \theta$ . Graph  $f$  below by either plotting points or using Desmos.

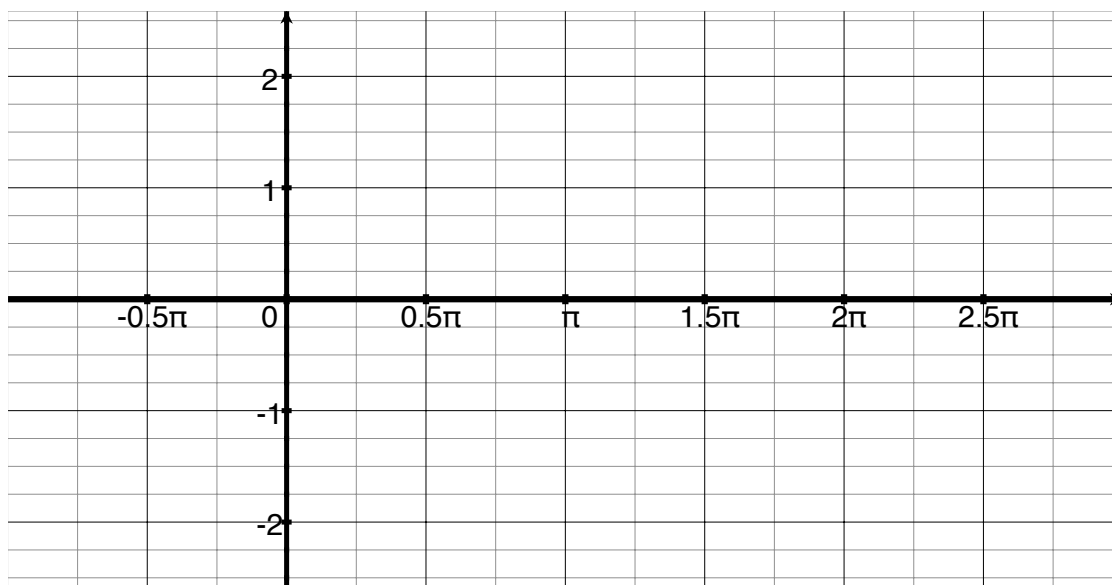


2. Recall section §1.5 (what was that about again?) and finish the following sentence:  
The graph of  $g(\theta) = -2 \cos(\theta) + 1.5$  looks like the graph of  $f$ , but...
3. Confirm #2 by using Desmos and draw the graph of  $g$  on the axis above.
4. Recall that the *amplitude* is (often defined as) half the difference in  $y$  values between the “peak” to the “trough”. Identify the amplitude of  $f$ , and  $g$ .
5. Functions are called periodic if the function “repeat themselves”. That is  $F$  is periodic if  $F(\theta) = F(\theta + k)$ . The smallest such constant  $k$  for a function is the period.
- Find the period of  $\cos \theta$
  - Find the period of  $\sin \theta$

6. Notice  $\sin(\theta)$  and  $\cos(\theta)$  make a complete wave in their respective periods. Think of  $\theta$  as time, then  $2\theta$  is rather like time running twice as fast. Thinking in these terms we see that  $\cos 2\theta$  completes its wave in half the time. Thus instead of taking  $2\pi$  to finish a period,  $\cos(2\theta)$  completes the wave in  $\frac{2\pi}{2}$  or  $\pi$ .

- What is the period of  $\sin(2\theta)$ ?

7. Note #6 implies the graph of  $\cos(2\theta)$  looks like the graph of  $\cos \theta$  but horizontally stretched by a factor of  $\frac{1}{2}$  (or compressed by a factor of two). Draw  $\sin(2\theta)$  by either plotting points, using Desmos, or reasoning.



8. Try to use the above reasonings to find the periods for the following functions:

- $\sin(3\theta)$ ?

- $3 \cos(\frac{1}{2}\theta)$ ?

9. Can you find the amplitudes for the functions in 8 without graphing them?

Verify 8 and 9's answers with Desmos

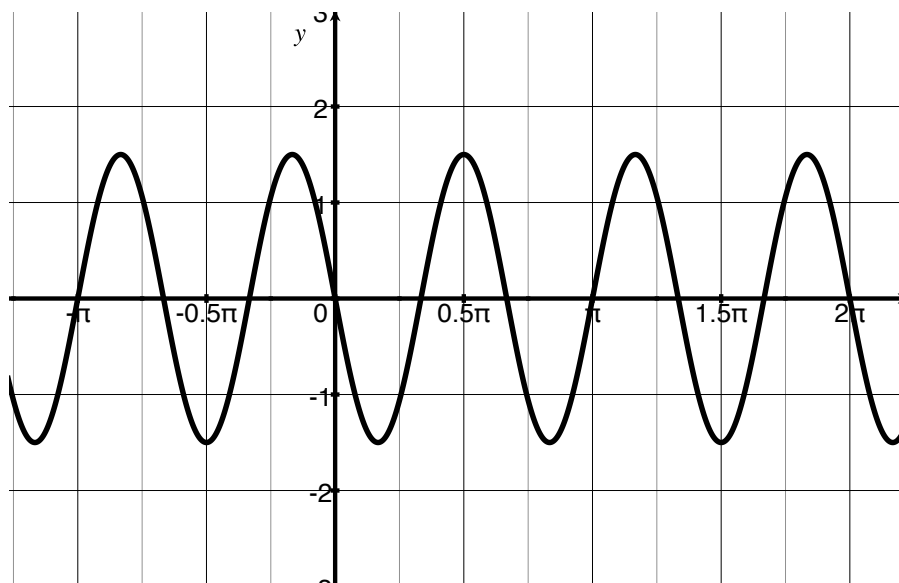
10. Draw the graph of  $y = a \sin(bx)$  where  $a > 0$  and  $b > 0$ .  
Consult the box on page 326 to check your answer.

11. Use your results from above to find the amplitude and period for the following functions.  
Verify you are correct by graphing each.

- $-2 \sin\left(\frac{1}{2}\theta\right)$

- $\cos\left(\frac{1}{2}\theta\right)$

12. Find the period and amplitude of the sine function below. Use the information you gathered in 10 to write an equation for the graph. Verify your answer with Desmos.



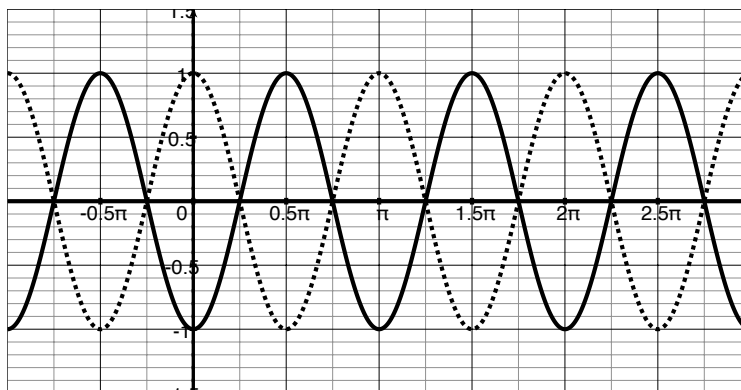
# Horizontal Stretches meet Horizontal Shifts

1. Consider  $\cos(2\theta + \pi)$ . The graph of this could look like the graph of  $\cos \theta$  but...

- |   |  |
|---|--|
| 1) shifted left $\pi$ units, and then,  | 1) horizontally stretched by $\frac{1}{2}$ so that |
| 2) horizontally stretched by $\frac{1}{2}$ so that the curve has period $\pi$ | or the curve has period $\pi$ , and then           |
|   | 2) shifted left $\pi$ units                        |

The two process above do *not* yield the same curve. In fact, the solid curve results from following the directions on the left and the dotted curve results from following the directions on the right.

Which one is the graph of  $\cos(2\theta + \pi)$ ?



2. Customarily (in computer engineering, and physics), people write  $\cos(2\theta + \pi)$  as  $\cos(2(\theta + \frac{\pi}{2}))$ . The number  $-\frac{\pi}{2}$  is called the phase shift. Explain why the number  $-\frac{\pi}{2}$  would be called “phase shift”.

3. Use the above to find the amplitude, period, and phase shift of  $\frac{3}{4} \cos\left(2\theta + \frac{2\pi}{3}\right)$ .