Polynomials

Definition 0.1. A *polynomial function of degree n* is a function of the form

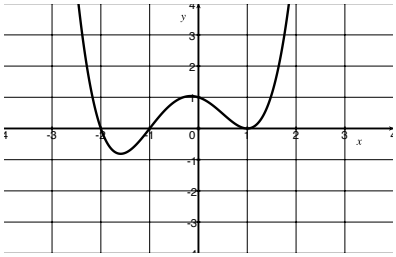
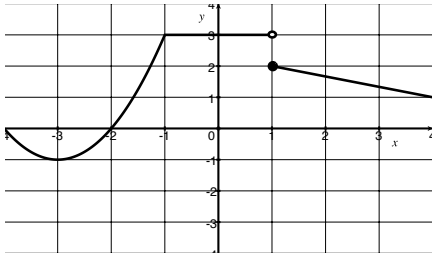
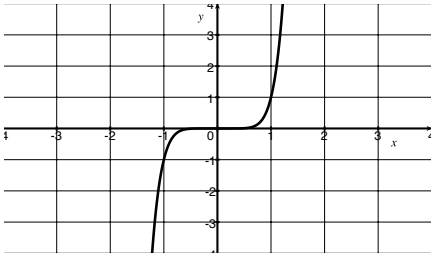
$$f(x) = a_n x^n + a_{n-1}x^{n-1} + ...a_2 x^2 + a_1 x + a_0$$

where n is a nonnegative integer and the coefficients a_n, a_{n-1}, $...a_2$, a_1, a_0 are real numbers with $a_n \neq 0$. The term $a_n x^n$ is called the *leading term*, the number a_n is called the *leading coefficient*, and a_0 is the *constant term*.

1. For each of the expressions below, determine if it is a polynomial, and *if it is*, determine the degree:

expression	polynomial? (yes/no)	leading term (if applicable)	degree (if applicable)
$117x^4 + 6x^{12} + x$			
$2^x - 5x^2$			
$\sqrt{5}x^2 - \pi$			
$7x^8 - 4.56x^4 - 7x^8 + x^2$			
3			
0			

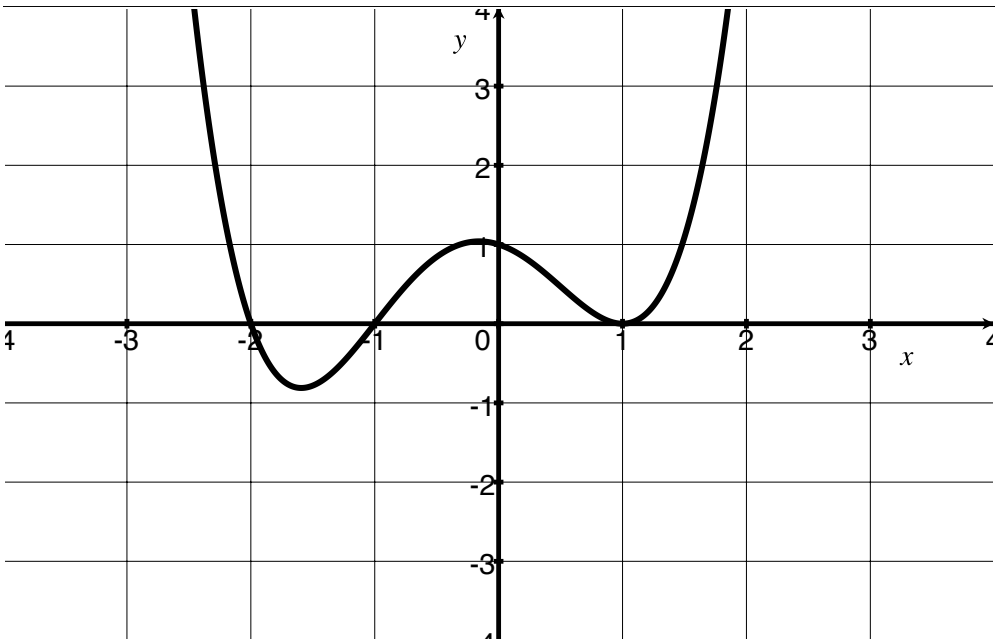
2. Which of the following could a polynomial?

3. For each graph above that could be a polynomial, determine what degree it could have.

Definition 0.2. A number c is called a *zero* of a function p if $p(c) = 0$. This number is also called a *root* or an *x-intercept*.

4. Given the graph belowis of a polynomial of degree four, find the algebraic rule.



Definition 0.3. A *rational function* r is a function of the form $r(x) = \dfrac{f(x)}{g(x)}$ where f and g are polynomials with g not equal to the zero function.

5. Perform the division: $\dfrac{2x^4 + x^3 - 16x^2 + 18}{x + 2}$

Note the above is example 1 on page 144 so you can check your work!