

# Inverses

While working in a group make sure you:

- Expect to make mistakes but be sure to reflect/learn from them!
- Are civil and are aware of your impact on others.
- Assume and engage with the strongest argument while assuming best intent.

**Definition 0.1.** The *inverse* of a function  $f$  is a function whose rule is

- $g(y) = x$  exactly when  $f(x) = y$ , or
- $g(f(x)) = x$  and  $f(g(y)) = y$ ,

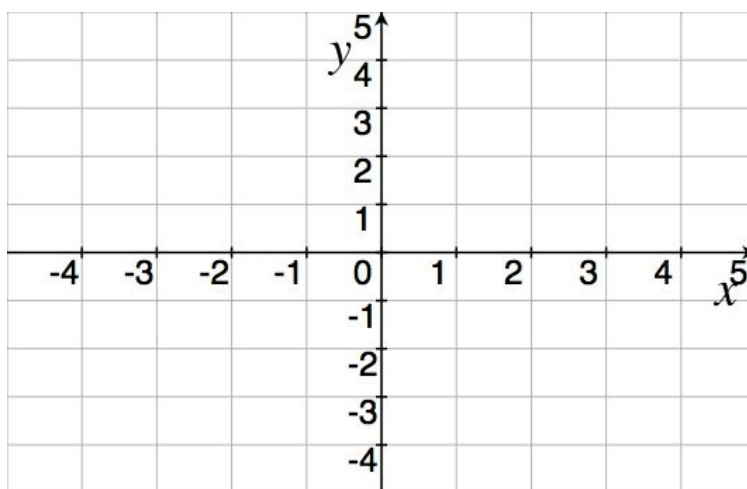
for all  $x$  and  $y$  in the respective domains.

1. Given that a tube partway filled with liquid will have a height  $h$ , dependent on the temperature,  $t$ .
  - (a) What does  $h(32) = 1$  mean in physical terms?
  - (b) What does  $h(212) = 10$  mean in physical terms?
  - (c) Describe the inverse function  $h^{-1}$  by identifying the inputs, outputs, and what we usually call this in real life.

**Horizontal Line Test 0.1.** A function has an inverse if and only if no horizontal line intersects the graph more than once. Functions that pass the horizontal line test are called one to one.

2. Let  $m$  be the function completely defined by the table:

$\star$	$m(\star)$	$\star$	$m^{-1}(\star)$
$-3$	$e$	$\sqrt{2}$	
$\frac{5}{2}$	$\sqrt{2}$	$e$	



- (a) Complete the table above to define  $m^{-1}$ .
- (b) Plot the graph of  $m$  on the set of axes provided.
- (c) Use a different mark/color to graph  $m^{-1}$  on the same axes.
- (d) Notice the point  $(\frac{5}{2}, \sqrt{2})$  is on the graph of  $m$  and  $(\sqrt{2}, \frac{5}{2})$  is on the graph of  $m^{-1}$ .
- (e) Find the domain of  $m$  and range of  $m^{-1}$ . Are there any similarities?
- (f) Find the range of  $m$  and the domain of  $m^{-1}$ . Are there any similarities?

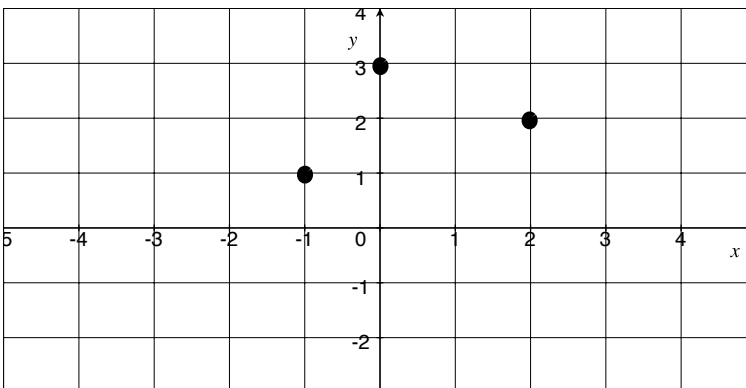
The observations you made in (e) & (f) are true in general, that is:

if  $f$  is the inverse of  $g$  then:      Domain of  $f$ =Range of  $g$       Range of  $f$ =Domain of  $g$

3. Let  $n$  be the function defined by the following graph:

(a) Will  $n$  have an inverse? Why?

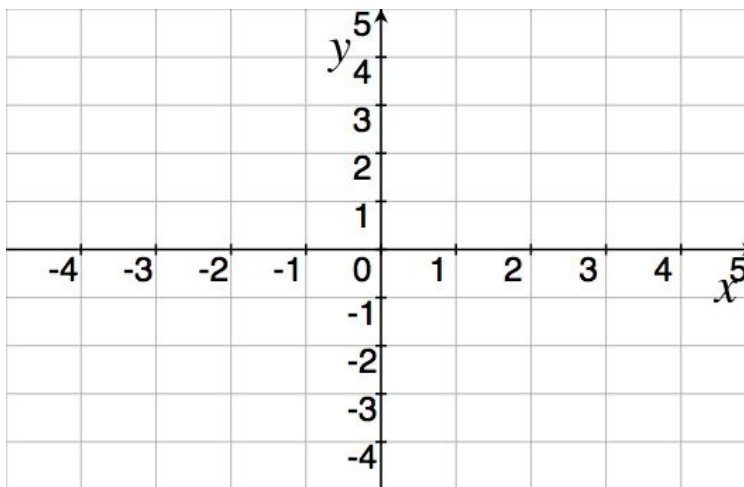
(b) Graph  $n^{-1}$ .



4. Let  $p$  be the function defined by  $p(x) = x^2 - 1$ .

(a) Draw the graph of  $p$ .

(b) Will  $p$  have an inverse? Why?



(c) What might we do to try and build something “kind of like an inverse”?