Midterm | TMath 221 | Spring 2017

/45

NAME: Key

1. [3] (4/4 Class) Explain how the square's dream in flatland about "The King" foreshadowed the remainder of the story.

(+1) related/from the book
(+1) what happened
(+1) how forshadowed

The square's relation to the King was mimiced by the sphere's relation to square. Both the square & the sphere were 1 dim higher then the individual they spoke to & both faced similar frustration as communicating with them

2. [3] (Weeks §4) Explain what strange behavior the surveyors of Flatland exhibited once returning home.

(+1) related/from the book
(+1) what happened
(+.5) true
(+.5) started.

The surveyors returned as a mirror image of their usual selves. Their right side became their left & vice versa. This means they did everything with the 'wrong' hand & had trouble reading 'normally'

3. [3] (HW3 #3) Group the following images into sets that have the same topology.

holes (+.5)
start (+.5)

same topology (+1)

own topology group (+.5)

own topology group (+.5)

4. [3] (4/20 Class) Identify or describe two "non-traditional" patterns or uses for origami.

start (+.5)
(+2) each one +1
clarity (+.5)

modular origami
↳ uses multiple sheets of paper
functional origami
↳ patterns that move

(from presentations)

heart stint
modeling airbags
getting objects into space

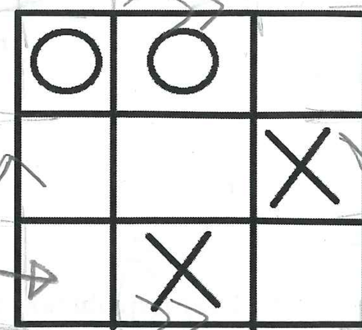
(from TED video)

5. [3] (Weeks §4) A tic-tac-toe board being played on a Klein Bottle is shown to the right. The game was started by X and now it is X's turn.
What is X's best move? Justify your choice.

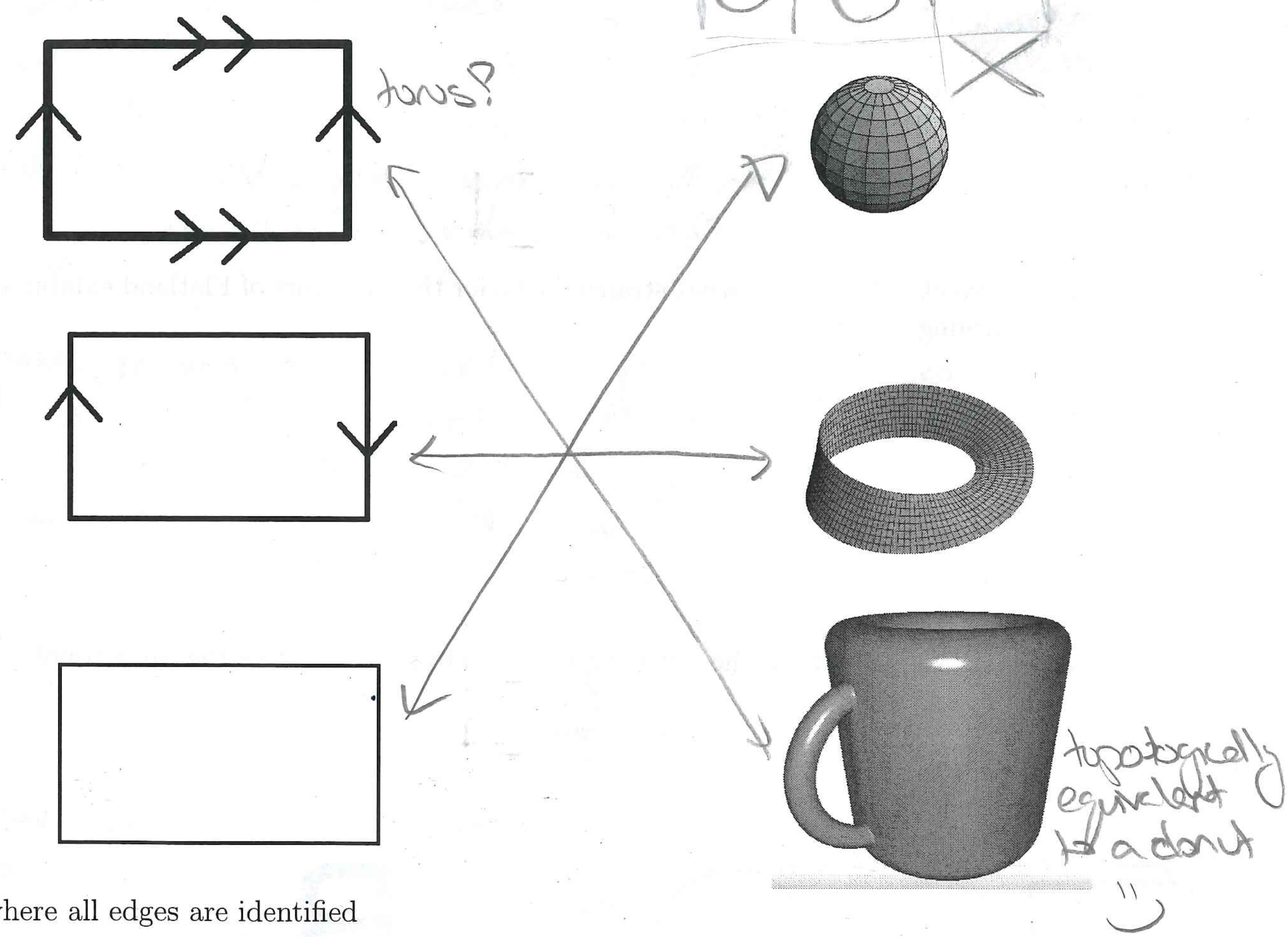
use gluings (+1)

(+1) [The shaded box would guarantee a win. This corresponds to

(+1)



6. [6] Match the items on the left to items with the same topology on the right.



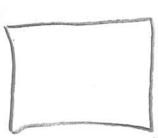
where all edges are identified

7. [3] (Weeks §3) Find a non-homogeneous 1 manifold.

(+1) ↳ 1 dimensional so looks like $\mathbb{R}$

(+1) not all points act like each other geometrically

Start (+.5)
got one (+.5)



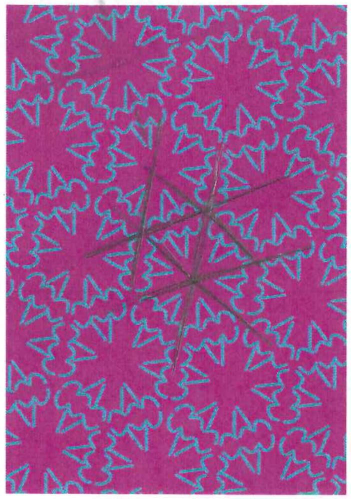
(only the lines - not the inside)

line segment b/c ends are strange/diff than the rest

↳ corners are strange

8. [4] (HW2 #1) Find the signature for each of the following.

(+1) {*333

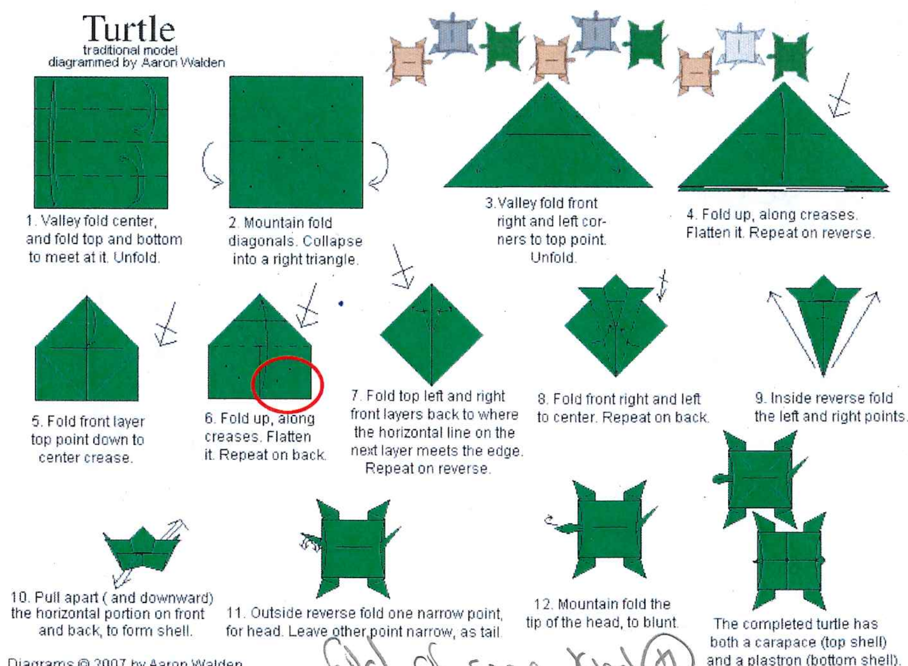


Cost
$1+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}$
$=2$ ✓
(+1)

(+1) {4*2

Cost
$\frac{3}{4}+1+\frac{1}{4}$
$=2$
(+1)

9. [2] (4/18 Class) Examine the Origami instructions below. Describe the kind of fold circled in the instructions.



fold of some kind (+1)

Mountain fold? (+1)
When the paper sits on the table the fold is higher then the edges - it make a "mountain" shape.

10. [5] (Class 4/11 & HW3 #1) Identify all possible signatures of a two dimensional tiling that includes only blue symbols and the symbols 2. Be sure to explain clearly *how* you know you have found all the possibilities. (You should note any theorems you use!)

Symbol	Cost ($)	Symbol	Cost ($)
○	2	* or ×	1
2	$\frac{1}{2}$	2	$\frac{1}{4}$
3	$\frac{2}{3}$	3	$\frac{2}{6}$ or $\frac{1}{3}$
4	$\frac{3}{4}$	4	$\frac{3}{8}$
5	$\frac{4}{5}$	5	$\frac{4}{10}$ or $\frac{2}{5}$
6	$\frac{5}{6}$	6	$\frac{5}{12}$
…	…	…	…
n	$\frac{n-1}{n}$	n	$\frac{n-1}{2n}$

Start (+1)

Magic theorem (+.5)

only blue (+.5)

Symbol 2 (+.5)

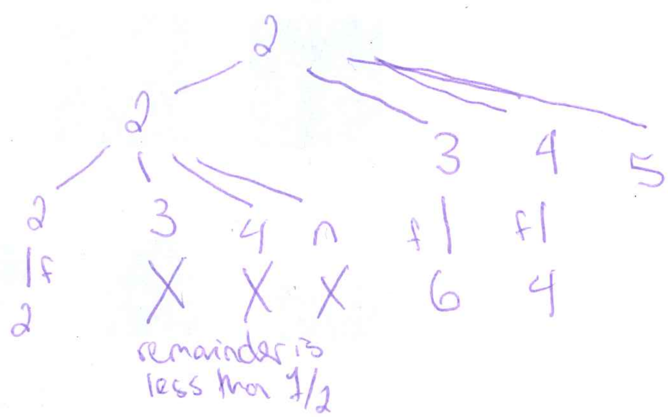
explain f (+1)

why can stop looking at 2n when $n \geq 7$. (+1)

possibilities:

2222 236 244 (+.5)

(done in class - see notes)



Midterm Group TMath 221 Spring 2017

NAMES:
A single copy of this problem can be turned in per group if interested.

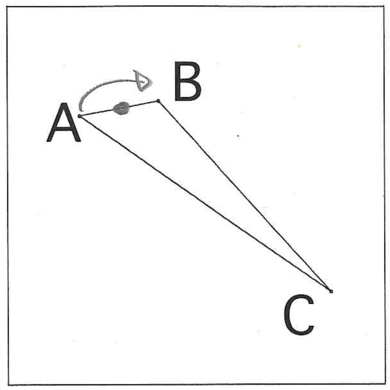
Halving the Area of a Triangle

start (+.5)
clarity (+3)
works for all Δ's (+3)
works for 1 Δ (+.5)
justification (+3)

1. [10] Given a triangle ABC on a patty paper, find a triangle inside $\triangle ABC$ that is half the area of $\triangle ABC$. Explain your process and *justify* why your method works. Make sure that your method works with all kinds of triangles and not just on special ones.

Hint: the area of a triangle is $\frac{1}{2} \cdot \text{base} \cdot \text{height}$.

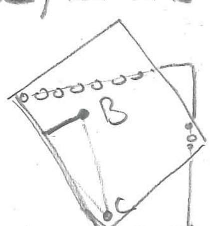
This is a patty paper exercise so the only tools you may use are patty paper(s) and a pencil.



Note: the below is not the only possible solution

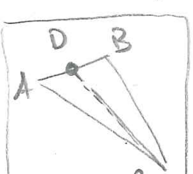
Since the area of a Δ is ½·base·height we can find a Δ with half the area by either halving the base (to get an area of $\frac{1}{2}(\frac{1}{2}\text{base})\cdot\text{height}$) or by halving the height (to get an area of $\frac{1}{2}\cdot\text{base}\cdot(\frac{1}{2}\text{height})$). Note in both situations the area is ½ of the original.

We will focus on halving the base. Given ΔABC, let $\overline{AB}$ be the base. Fold the line $\overline{AB}$ back on itself so that point A lies on top of point B. (Fold shown on above diagram)



Unfold. Now we have a point between A & B on the line AB made by the fold, label this point D. Use another patty paper to draw a straight line from D to C.

dotted lines are edges of lower layer



The Δ ADC has half the area of ΔABC because the base AD is half the length of AB. (When folding $\overline{AB}$ over itself we guaranteed the segments AD = DB.)

Note also that BDC is also half the area of ΔABC