Final TMath 221 /50 Spring 2016

NAME: Key

1. [2] (Weeks §5) A. Square began a movement in *The Shape of Space* to figure out the kind of universe Flatland was. What was the conclusion?

a torus (+1) connect sum with a projective plane (+1)

so $T^2 \# \mathbb{P}^2$

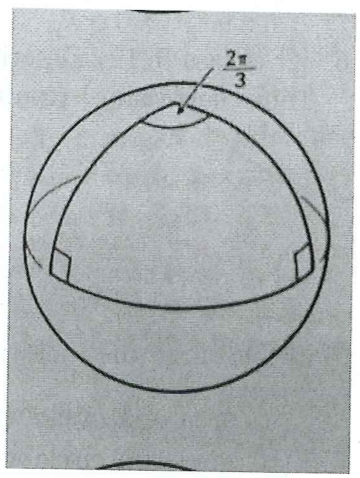
2. [2] (§5 #1) What do you get when you form the connected sum of a six-holed doughnut surface and a eleven-holed doughnut surface?

a seventeen-holed doughnut

$6+11=17$

Know connect sum (+1) got it (+1)

3. [3] (HW5 #4) Find the area of the triangle to the right given that the sphere has radius 1 (which we usually assume!)



Area of Δ on S^2

$\pi \cdot (\text{sum of angles}) - \pi$

$\pi \cdot \left(\frac{\pi}{2} + \frac{\pi}{2} + \frac{2\pi}{3}\right) - \pi = \frac{2\pi}{3} \text{ units}^2$

or 120

radian/deg conversion (+1)
formula (+1)
got it (+1)

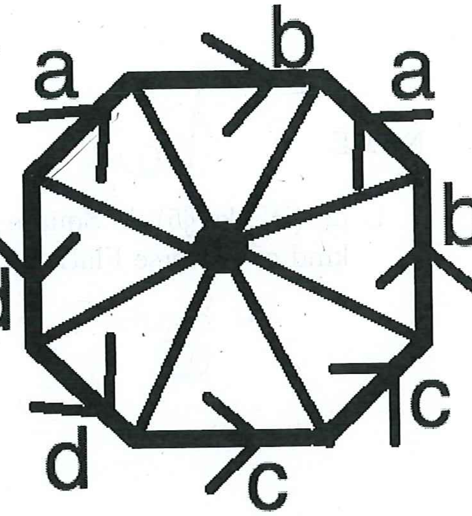
4. [3] (HW4 #7) Explain how to make a geodesic parallel to the equator on a sphere.

Trick Question?

Geodesics on a sphere are all great circles. (+1)

Any great circle will intersect (+1) the equator thus we cannot (+1) make a geodesic parallel to the equator

5. (Euler Practice Wks) Consider the object shown to the right.



(a) [3] Identify the surface using only $\mathbb{P}^2$, $\mathbb{T}^2$, and #'s.

$\mathbb{T}^2 \# \mathbb{P}^2 \# \mathbb{P}^2$

or $\mathbb{P}^2 \# \mathbb{P}^2 \# \mathbb{P}^2 \# \mathbb{P}^2$ or $\mathbb{K}^2 \# \mathbb{P}^2 \# \mathbb{P}^2$

(b) [1] (HW6 #2) Find another way to denote the surface shown.

see above

(c) [1] Find the number of edges.

$\underset{(a)}{1} + \underset{(b)}{1} + \underset{(c)}{1} + \underset{(d)}{1} + \underset{(\text{internal})}{8} = 12$

(d) [2] Compute the Euler number χ

(+1) Vertices − edges + faces

$2 - 12 + 8 = -2$

6. [2] (Class 5/19) Describe an 'application' of the Euler number. That is, what (sometimes surprising) results follow once we know the Euler number?

the brussel's sprouts game has no strategy but is completly determined by who goes first | or | the # of hexagons needed to tile a sphere is completly determined by the sphere's Euler number of 2

7. [4] Arrange the following from largest to smallest? Explain your reasoning.

(a) area of a circle with radius 2 in $\mathbb{E}^2$

(b) area of a circle with radius 1 in S^2

(c) area of a circle with radius 2 in $\mathbb{H}^2$

(c) area of circle with radius 2 in $\mathbb{H}^2$ > (a) area of circle with radius 2 in $\mathbb{E}^2$ > (b) area of a circle with radius 1 in S^2) order (+2)

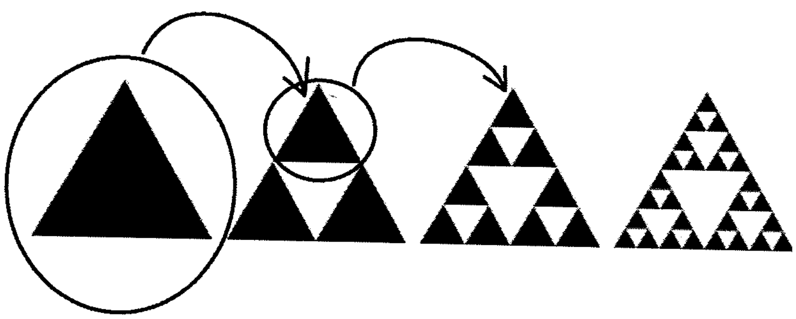
(+1) both circles have the same radius, However in $\mathbb{H}^2$ we have too much area to flatten out a circle onto one in $\mathbb{E}^2$. There would be excess fabric

(+1) If we tried to flatten out the circle w/ radius 1 in S^2 onto a circle in $\mathbb{E}^2$, the area would tear b/c there is less area in the circle in S^2 then one w/ radius 1 in $\mathbb{E}^2$. So certainly the area is less than a radius 2 in $\mathbb{E}^2$

(+2)

8. [3] (Class 5/24) *Carefully* and *precisely* define an object with dimension greater than one but not (topologically) equivalent to a (filled in) square.

(+1) Sierpinski's Carpet or Koch curve or Dragon Curve



9. [3] (Fractals Wks) Sierpinski's triangle is shown above. Find the fractional (Hausdorff) dimension. Clearly show your steps!

(+1) Scale : 1:2

(+1) use 3 versions of smaller △

(+.5) $2^{Dim} = 3$

$\log 2^{Dim} = \log 3$

$Dim \log 2 = \log 3$

$Dim = \frac{\log 3}{\log 2} \approx$

(+.5)

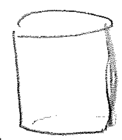
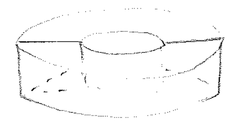
10. [6] Choose only *TWO* of the following. Clearly identify which of the two you are answering and what work you want considered for credit. Answering more questions can earn you up to 4 points extra credit.

   (a) (Product Spaces Talk) Describe geometrically the product of a one dimensional interval and a one dimensional circle. Describe topologically the product of a one dimensional circle and a one dimensional square. (By one dimensional I meal the 'edges' of the object and am not including the 'inside'.)

   (b) (Hypersphere Talk) Consider the hypersphere S^3. What dimensional space does S^3 rest in? What dimension is S^3? Explain how these two questions are different.

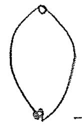
   (c) (Hyperbolic Space Talk) Consider a baseball field in Euclidean Space and one in Hyperbolic Space. Which field has more area? Which field is more likely to hold games where no one makes a home run? Explain your reasoning.

   (d) (Universe Talk) Identify two hypothesis that scientists have found to describe the 'life cycle' of the universe that fits with observable data. Clearly indicate where the two hypothesis differ.

a) product of interval and circle: right cylinder 
topologically circle and square: equivalent to a torus 

b) S^3 rests in 4 dimensional space
S^3 is 3 dimensional
Objects sometimes 'don't fit' in lower dimensional spaces. For example the Klein bottle is a 2 dimensional object but needs to rest in 4 dimensions for there to be no self intersections.

c) The baseball field in hyperbolic space has more area than the one in E^3
The Euclidean field is more likely to have games where no one scores. In hyperbolic space the pitcher must be much more precise to get the ball into the strick zone. Most batters will walk leading to very high scoring games.

d) i)  → big crunch → life → big bang (+1)

ii)  repeated big bang and big crunch (+4)

iii)  no singularities but there is a repeated pulling apart & pushing together

differences (+1) iv) big rip

Final Group

TMath 221

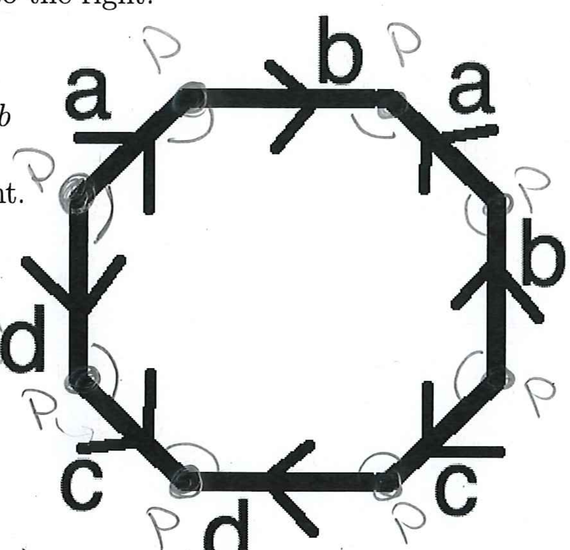
Spring 2016

NAMES: Key

Polygons in Strange Spaces

1. [2] Identify the topological surface shown to the right.

   $\mathbb{T}^2 \# \mathbb{T}^2$



2. [6] Consider the point between side a and b (which is also the point between c and d). Determine the total angle around this point. Justify your computations.

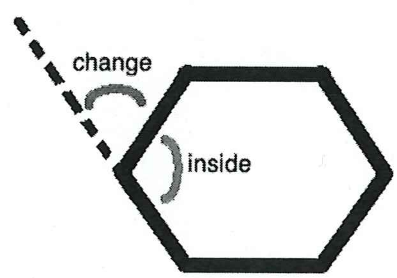
Notice that the angle around point P will be the sum of the "inside" angles of the octagon. From a worksheet done in class we know the sum of angles in an octagon are $180°(8-2) = 180°\cdot 6 = 1080°$ (regular or not) (+1)

variables (+1) if regular octagon (+1)
understand? (+1) reasoning/why (+2)

3. [2] Deduce if this surface has Euclidean, Spheric

   hyperbolic

Recall that we can define the "change" angle
be the angle of rotation
when pivoting or changing from one side to
another as we travel clockwise around the polyg



4. [5] Find the sum of "change" angles for a polygon with n sides in this space. *Justify* your answer on the back of this sheet.

1080° (+1) b/c corresponds to the total angle around one point (+1)
variables (+1) reasoning why (+2)