Creative Educational Experience

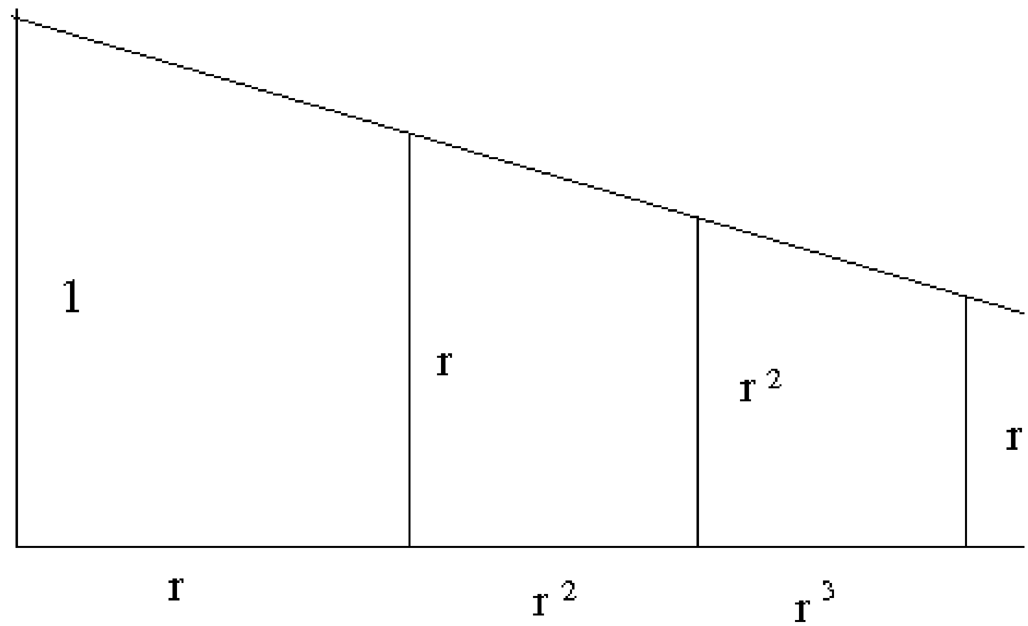
Topic: Mathematics
Title: A Finite, Infinite Sum ($6^{\text{th}} - 12^{\text{th}}$ grade)
Presenter: Ruth Vanderpool Ph.D. Lecturer at University of Washington Tacoma
Activity: This activity runs about 40 minutes and builds the geometric sequence $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + ...$ with paper in such a way that students can see that the infinite sum converges to one. Older audiences that are comfortable with lines ($9^{\text{th}} - 12^{\text{th}}$ grade) can verify their conclusions with a historical proof presented in Hairer & Wanner's *Analysis by Its History*. Further consideration will be given to additional geometric sequences such as $\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + ...$ and perhaps students can discover $\sum_{n=1}^{\infty} \left(\frac{1}{r}\right)^n = \frac{1}{r-1}$.

Lecture Notes:

Provide everyone with 1ft x 1ft square. Have them fold each of the corners to the middle and color the flaps red. We'll need to figure out the area of the red square, perhaps by noticing the red covers half of the original square. Put the white side down and red side up and again fold the corners to the middle of the paper completely covering the red that was colored. Color the flaps that now show orange and again find the area of the orange square by using rulers or the same thought experiment as before. Start a table on the board recording next to each color the length of the side and the area. Do the procedure again, but color the flaps yellow. Let them try to complete the rainbow. After they get to the point that they can't fold the square anymore see if they can figure out the pattern of the side lengths. See if we can figure out the pattern of the area lengths. Now, ask them what all the areas added up would be. We can see it geometrically by unfolding the paper and noticing all the colors are on one side and completely make up the 1ft x 1ft square.

If past algebra:

- Introduce the idea of infinite sums & work through $\sum_{n=1}^{\infty} \frac{1}{2^n}$ with paper.
- Rework $\sum_{n=1}^{\infty} \frac{1}{2^n}$ using the triangle method & comment on Zeno's paradox
- Let them work on $\sum_{n=1}^{\infty} \frac{1}{3^n}$ with the triangle method & then review.
- Show them $\sum_{n=1}^{\infty} \frac{1}{r^n}$
- Possible extension: consider $\sum_{n=1}^{\infty} a\frac{1}{r^n}$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