# **Observable consequences of planet formation models in systems with close-in terrestrial planets**

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# ABSTRACT

To date, two planetary systems have been discovered with close-in, terrestrial-mass planets  $(\leq 5-10 \,\mathrm{M_{\oplus}})$ . Many more such discoveries are anticipated in the coming years with radial velocity and transit searches. Here we investigate the different mechanisms that could form 'hot Earths' and their observable predictions. Models include: (1) in situ accretion; (2) formation at larger orbital distance followed by inward 'type 1' migration; (3) formation from material being 'shepherded' inward by a migrating gas giant planet; (4) formation from material being shepherded by moving secular resonances during dispersal of the protoplanetary disc; (5) tidal circularization of eccentric terrestrial planets with close-in perihelion distances and (6) photoevaporative mass-loss of a close-in giant planet. Models 1-4 have been validated in previous work. We show that tidal circularization can form hot Earths, but only for relatively massive planets ( $\gtrsim 5 M_{\oplus}$ ) with very close-in perihelion distances ( $\lesssim 0.025$  au), and even then the net inward movement in orbital distance is at most only 0.1-0.15 au. For planets of less than  $\sim$ 70 M<sub> $\oplus$ </sub>, photoevaporation can remove the planet's envelope and leave behind the solid core on a Gyr time-scale, but only for planets inside 0.025–0.05 au. Using two quantities that are observable by current and upcoming missions, we show that these models each produce unique signatures, and can be observationally distinguished. These observables are the planetary system architecture (detectable with radial velocities, transits and transit timing) and the bulk composition of transiting close-in terrestrial planets (measured by transits via the planet's radius).

**Key words:** astrobiology – methods: *N*-body simulations – methods: numerical – planetary systems: formation – planetary systems: protoplanetary discs.

# **1 INTRODUCTION**

Both radial velocity (RV) and transit searches are biased toward finding large/massive planets at small orbital distances (e.g. Marcy & Butler 1998; Charbonneau et al. 2007). Given the increased sensitivity of new instruments, ever-smaller close-in planets are being detected. Currently, two systems are thought to contain close-in planets of less than 10 Earth masses ( $M_{\oplus}$ ): GJ 876 (Rivera et al. 2005) and GJ 581 (Udry et al. 2007). Transit missions *CoRoT* (Baglin 2003) and *Kepler* (Basri, Borucki & Koch 2005) expect to find perhaps a few hundred close-in planets with masses less than 5–10  $M_{\oplus}$ . In this paper we focus on these 'hot Earth' planets, which we assume

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to have masses in the range  $0.1 < m_{\rm p} < 10 M_{\oplus}$ , and semimajor axes  $a \lesssim 0.2$  au.

We propose that it is possible to determine the formation history of a given hot Earth planetary system with two observable quantities: the architecture of the inner planetary system, and the bulk composition of the hot Earth(s). The planetary system architecture can be detected by a combination of RV and transit measurements, as well as additional analysis of transit signals (e.g. transit timing variations: TTV; Agol et al. 2005; Holman & Murray 2005). The composition of a transiting terrestrial planet can be determined by its physical size, i.e. the transit depth. Structure models indicate that very waterrich planets ( $\gtrsim$ 10 per cent water by mass) have detectably larger radii than dry, rocky planets or iron-dominated planets (Fortney, Marley & Barnes 2007; Seager et al. 2007; Sotin, Grasset & Mocquet 2007; Valencia, Sasselov & O'Connell 2007a,b), although a massive H/He envelope can also inflate the observed planetary radius (Adams, Seager & Elkins-Tanton 2007).

Table 1. Observable predictions	of hot Earth formation models.
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Model	System architecture	Planet composition
In situ accretion	Several hot Earths, spaced by ~20–60 mutual Hill radii	Relatively dry for solar-type stars. Up to 0.1–1 per cent water for low-mass stars.
Type 1 migration	Chain of many terrestrial planets, close to mutual MMRs	Icy or rocky, depending on formation zone. Most likely to be icy $(\gtrsim 10 \text{ per cent water by mass}).$
Giant planet migration shepherding	Coexistence of hot Earths and close-in giant planets near (but not in) strong MMRs	Rocky with moderate water content: a few per cent water by mass at time of formation.
Secular resonance shepherding during disc dissipation	Coexistence of hot Earths and at least two, interacting giant planets	Depends on the details of the giant planet's orbital history. Rocky if formed mainly <i>in situ</i> .
Tidal circularization of eccentric planets	Single hot Earth, with possible distant companion (giant planet or stellar binary) to explain high eccentricity.	Depends on formation zone of planet – rocky unless migrated inward.
Photoevaporation of hot Neptunes	Hot Earth inside 0.025–0.05 au. Likely chain of several planets, as for type 1 migration. Correlation between hot Earth versus hot Neptune frequency and stellar age.	Icy, assuming what remains is a giant planet core.

Several mechanisms for the formation of close-in terrestrial planets have been proposed (Zhou et al. 2005; Gaidos et al. 2007). In Section 2 we describe the observable quantities that can distinguish between models. In Section 3, we summarize four known models, and test two unproven models: (i) tidal circularization of terrestrial planets on eccentric orbits and (ii) photoevaporation of hot Neptunes or hot Jupiters. We have tried to include all reasonable models, which include various combinations of accretionary growth, planet migration, and evaporative loss. Table 1 summarizes the observable differences between these models. In Section 4, we apply these models to the two known hot Earth systems. Section 5 concludes the paper with a discussion of whether the mechanism for giant planet formation – core accretion or gravitational instability – can affect the abundance of hot Earths, as claimed by Zhou et al. (2005).

# **2 OBSERVABLE QUANTITIES**

The observables considered in this paper are the architecture of the inner planetary system and the bulk planetary composition. The planetary system architecture, i.e. the coexistence (or lack) of additional planets in hot Earth systems, can provide strong circumstantial evidence for or against certain formation models, as described below. In particular, certain characteristic planetary configurations are smoking guns (see Table 1). Determining the bulk composition of a planet requires transit measurements. Thus, our analysis applies only to systems with at least one transiting planet. In most cases, but not all, the transiting planet must be a hot Earth.

The architecture of hot Earth planetary systems may be determined in three primary ways: (i) via the detection of transits of multiple planets; (ii) via RV monitoring of the host star and (iii) by analysis of TTV. Other techniques such as astrometry may be used in conjunction with these techniques, but note that astrometry is not optimal for detecting close-in planets (Black & Scargle 1982). Detection of multiple transiting planets in the same system requires extremely low mutual inclinations between planetary orbits, which are thought to be rare (e.g. Levison, Lissauer & Duncan 1998). The RV technique has discovered several planets with minimum masses less than Neptune, including the two known systems with hot Earths (Rivera et al. 2005; Udry et al. 2007). There exist several currently operational instruments capable of RV follow-up for *CoRoT* and *Kepler* targets, such as Keck HIRES (Vogt et al. 1994), the Hobby-Eberly Telescope's HRS spectrograph (Cochran et al. 2004), and the HARPS instrument at ESO (Mayor et al. 2003). In addition, the HARPS-North spectrograph is being built specifically to do RV follow-up of *Kepler* candidate transiting planets (Latham 2007). However, given that many of the target stars will be very faint, RV follow-up of a large number of stars may not be possible. For those that can be followed up, the  $\leq m s^{-1}$  sensitivity of current RV instruments should be able to detect close-in,  $\leq 5-10 M_{\oplus}$  planets and to probe the inner regions of *CoRoT* and *Kepler*-detected targets.

TTV analysis measures the deviation of a series of transits from a perfect chronometer, representing a deviation of the transiting planet's orbit from a perfect Keplerian ellipse due to perturbations from one or more additional planets (Agol et al. 2005; Holman & Murray 2005). The TTV signal scales with the transiting planet's orbital period, and increases for more massive and closer perturbing planets. For sufficiently accurate transit timing data, TTV analysis can either derive the mass and orbit of a perturbing planet or place constraints on the existence of nearby perturbers (Steffen & Agol 2005; Agol & Steffen 2007). TTV is especially sensitive to planets that lie in or close to mean motion resonances (MMRs) with the transiting planet, which is convenient given that several formation models predict near-resonant planetary configurations (see Section 3 below).

The bulk composition of a planet determines its density and therefore its physical size: ice planets are far larger than iron planets. Recently, several studies have calculated mass-radius relations for planets with different compositions (Valencia, O'Connell & Sasselov 2006; Fortney et al. 2007; Seager et al. 2007; Sotin et al. 2007; Valencia et al. 2007a,b). For a fixed mass, there exists a roughly 40 per cent difference in radius between pure ice planets and pure rock planets, and a similar 40 per cent difference between pure rock and pure iron planets; these ratios of sizes are independent of planet mass. In addition, there is a  $\sim$ 35 per cent difference in size

between Earth-like planets (2/3 rock, 1/3 iron) and ocean planets (1/2 rock, 1/2 water; Fortney et al. 2007).<sup>1</sup>

# Estimates of both the planetary mass and radius are needed to derive a bulk composition (Selsis et al. 2007). For transiting planets, errors in stellar masses and radii (Ford, Rasio & Sills 1999; Cody & Sasselov 2002; Fischer & Valenti 2005; Sozzetti et al. 2007) are likely to lead to errors in planetary radii of the order of 2–10 per cent (see section 6 of Seager et al. 2007). With a determination of the planetary mass to within 5 per cent and radius to within 10 per cent, it may be possible to differentiate between mostly rocky (Earth-like) planets and icy planets with $\geq$ 10 per cent water by mass (Valencia et al. 2007b). For transiting planets with very precise mass and radius measurements (better than 2 per cent), more detailed compositions may be derived (Seager et al. 2007).

Atmospheric envelops of H/He can inflate the observed radii of solid planets by tens of per cent (Adams et al. 2007). For a given radius measurement, solutions for the planetary structure become degenerate with respect to water content and envelope mass: small radii are clear signatures of rocky planets, but larger radii are ambiguous. Theoretical models suggest that the ability of a planet to accrete a gaseous envelope depends on the planet's mass, and is not sensitive to the orbital distance (e.g. Ikoma, Emori & Nakazawa 2001; Ida & Lin 2004).<sup>2</sup> Thus, additional information about the presence and thickness of the planet's atmosphere is needed to determine whether the planet is water-rich or rocky. For example, more information could be gathered from a situation in which an atmosphere has almost certainly photoevaporated away (old stellar age plus very close-in planet). Another favourable case would be a situation for which some spectral information about the planet's atmosphere could potentially be obtained.

Observational limitations are such that certain planets will have no mass estimates, given the faintness of their host stars and the consequent difficulty of RV follow-up. For such cases, it is possible to place mass limits based on maximum and minimum radius estimates, i.e. by assuming the planet to be made of pure iron or pure water (or pure hydrogen for gaseous planets). In addition, the bulk planetary composition may not be determined in many cases because of observational limitations (Selsis et al. 2007) or degeneracy between model parameters (Adams et al. 2007). With no composition information it becomes more difficult to differentiate between formation models. None the less, several cases can be distinguished if the inner planetary system architecture is known.

Thus, current and future programmes have the sensitivity to determine the orbits, masses, radii and companions of a large number of hot Earths. Although it will not be feasible in all cases, information about other planets in hot Earth systems will be determined via RV, transits and transit timing analysis. In this paper we focus on systems in which both the inner planetary architecture and the composition of a hot Earth (rocky versus >10 per cent water) can be determined (i.e. the brightest *CoRoT* and *Kepler* targets; see fig. 6 of Selsis et al. 2007). As explained below and summarized in Table 1, analysis of these data may be able to identify the formation mechanism of such planets.

# **3 MODELS FOR HOT EARTH FORMATION**

Here we investigate six mechanisms for hot Earth formation, including proven and previously untested mechanisms. The six models are: *in situ* accretion (Section 3.1); type 1 migration (Section 3.2); shepherding during giant planet migration (Section 3.3); shepherding via secular resonance (SR) sweeping (Section 3.4); tidal circularization of eccentric planets (Section 3.5) and photoevaporation of close-in giant planets(Section 3.6). For each model, we discuss the state of the inner planetary system, as well as the likely composition of the hot Earth(s). Table 1 summarizes the differences between models. The first four models listed have been demonstrated in previous work. We introduce two additional models for hot Earth formation, and test them quantitatively below.

# 3.1 In situ formation

If protoplanetary discs contain a substantial mass in solids close to their stars, then perhaps hot Earths can form from local material. This depends critically on the condensation temperatures of grains (Pollack et al. 1994; Lodders 2003), discs' inner truncation radii (e.g. Akeson et al. 2005; Eisner et al. 2005), and the surface density profile of solids (Weidenschilling 1977; Hayashi 1981; Davis 2005; Raymond, Quinn & Lunine 2005). If hot Earths form *in situ*, then their growth would be similar to that of Solar system's terrestrial planets (Wetherill 1990, 1996; Chambers & Wetherill 1998; Agnor, Canup & Levison 1999; Morbidelli et al. 2000; Chambers 2001; Kenyon & Bromley 2006), but minus the dynamical effects of Jupiter and Saturn (although giant planets may coexist with some hot Earths, e.g. Gliese 876; Rivera et al. 2005).

If there is sufficient mass to form one hot Earth in situ, then we expect a population of several hot Earths to form, with masses determined by the local disc mass and spacings similar to those in the Solar system and in accretion simulations (roughly 20-80 mutual Hill radii:  $R_{\rm H,m} = 0.5(a_1 + a_2) (M_1 + M_2/3 M_{\star})^{1/3}$ ;  $a_1$  and  $a_2$  are the orbital radii and  $M_1$  and  $M_2$  the masses of two adjacent planets). The surface density distribution of protoplanetary discs,  $\Sigma$ , is thought to scale with radial distance r as  $\Sigma \sim fr^{-\alpha}$ , where f is a scalefactor to account for the large variability in observed disc masses (Andre & Montmerle 1994; Eisner & Carpenter 2003; Andrews & Williams 2005; Scholz, Jayawardhana & Wood 2006), and the value of  $\alpha$  lies between 0.5 and 2 (Weidenschilling 1977; Hayashi 1981; Kuchner 2004; Davis 2005; Andrews & Williams 2007; Dullemond et al. 2007; Garaud & Lin 2007). Accretion models suggest that planets close to their stars are generally smaller than those farther out for  $\alpha < 2$  (Lissauer 1987; Kokubo & Ida 2002; Raymond et al. 2005; Kokubo, Kominami & Ida 2006; Raymond, Scalo & Meadows 2007).

For solar-type stars, hot Earths that form *in situ* are likely to be dry because of the low efficiency of water delivery from both comets (Levison et al. 2000) and asteroids (Raymond, Quinn & Lunine 2004). Because of the very hot local temperatures, these planets would be mainly composed of refractory materials such as iron and rock (Pollack et al. 1994; Lodders 2003). However, for the case of low-mass stars, the snow line is located very close-in (as is the habitable zone – Kasting, Whitmire & Reynolds 1993). Water delivery to hot Earths may therefore be more favourable around low-mass stars, although impact speeds are high and formation times fast compared with Earth's formation zone (Lissauer 2007; Raymond et al. 2007), and the snow line moves significantly in the disc lifetime (Sasselov & Lecar 2000; Kennedy, Kenyon & Bromley 2006).

<sup>&</sup>lt;sup>1</sup> Note that water contents of a few per cent by mass, though  $\gtrsim 10-20$  times larger than the Earth's estimated water budget (Lécuyer, Gillet & Robert 1998), would have a negligible effect on the planetary radius compared with the observational uncertainties.

 $<sup>^2</sup>$  In addition, as Adams et al. (2007) point out, significant envelopes of hydrogen may be formed as a result of outgassing from the planetary interior.



**Figure 1.** Snapshots of *in situ* accretion of a system of hot Earths. Each object's size is proportional to mass<sup>1/3</sup>, and the colour corresponds to its water content, from  $10^{-5}$  water by mass (red online) to 5 per cent water by mass (darkest blue online; see colour bar). The dark circle in the centre of each body refers to the relative size of its iron core (see Raymond et al. 2005 for details). In this case, the central star is  $0.31 M_{\odot}$ , the same as Gliese 581.

Fig. 1 shows snapshots of in situ accretion of terrestrial material close to a 0.31 M<sub>☉</sub> star from Raymond et al. (2007, in preparation), designed to examine the GJ 581 system. The simulation started from a disc of 57 planetary embryos (initially separated by 3-6 Hill radii, as in Raymond, Quinn & Lunine 2006a) and 500 planetesimals in a very massive disc totalling  $40\,M_{igoplus}$  between 0.03 and 0.5 au. The disc's surface density decreased with orbital distance r as  $r^{-1}$  (i.e.  $\alpha = 1$ ), and was roughly 30 times more massive than the minimummass solar nebula (MMSN) model (Weidenschilling 1977; Hayashi 1981; Davis 2005).<sup>3</sup> The three planets that formed in this simulation have masses of  $6.6 \,\mathrm{M}_{igodot}$  (at 0.06 au),  $10.9 \,\mathrm{M}_{igodot}$  (0.12 au) and  $10.6 \,\mathrm{M}_{\oplus}$  (0.30 au). Each has a substantial water content, acquired via collisions with material originating beyond the 'water line' at 0.29 au, but note that the effects of water depletion during impacts (Genda & Abe 2005; Canup & Pierazzo 2006) and hydrodynamic escape (Matsui & Abe 1986; Kasting 1988) have not been accounted for.

Thus, if hot Earths form *in situ* then we expect systems of several hot Earths to form, with spacings comparable to the Solar system terrestrial planets. These planets will contain mainly local, dry material, although for low-mass stars they may contain up to perhaps 0.1–1 per cent water by mass (but see Lissauer 2007; Raymond et al. 2007).

# 3.2 Inward Type 1 migration

Planets more massive than roughly a Mars mass excite density waves in the disc (Goldreich & Tremaine 1979). The back reaction of these waves on the planet causes inward orbital migration on a time-scale of 10<sup>4</sup>–10<sup>6</sup> yr (Goldreich & Tremaine 1980; Ward 1986, 1997; D'Angelo, Kley & Henning 2003; Masset, D'Angelo & Kley 2006a). However, type 1 migration may be stopped or even reversed in the very inner regions of the disc, because of net disc torque changes at disc edges (Masset et al. 2006b) or in the optically thick inner disc (Paardekooper & Mellema 2006). Thus, hot Earths could form far from their stars and migrate in to the location in the disc where the disc torques cancel out and migration is stopped. Presumably, if there is enough solid and gas mass to enable type 1 migration of one planet, then others should follow. Comigrating planets may end up trapped in MMRs (e.g. Lee & Peale 2002), and can form chains of many planets in paired resonances. These orbital chains of planets can survive for long times, as the outward-directed torques on the inner planets may be balanced by inward-directed torques on the outer planets (Terquem & Papaloizou 2007). Surviving planets do not remain on strictly resonant orbits, and collisions between planets can occur after the disc dissipates.

The amount of solid material in the disc is thought to increase by a factor of 2–4 or perhaps more beyond the snow line (Hayashi 1981; Stevenson & Lunine 1988; Lodders 2003). In addition, most disc surface density profiles contain far more mass in their outer regions. Thus, it seems reasonable to assume that, in this model, most hot Earths must form beyond the snow line and are therefore icy in composition rather than rocky. Indeed, transits of the hot Neptune GJ 436 b have been interpreted as an indication that it may be largely composed of water and may therefore have formed beyond the snow line and migrated inward (Gillon et al. 2007). Note, however, that inferring a detailed planetary composition from a radius measurement is ambiguous because different combinations of rock, ice and H/He envelopes can form planets with the same mass and radius (see fig. 3 from Adams et al. 2007). In addition, formation and migration models suggest that in a system of several

<sup>&</sup>lt;sup>3</sup> Note that this discs as massive as the one in Fig. 1 (30 times the minimummass disc) are likely to be quite rare (e.g. Andrews & Williams 2005). Thus, *in situ* accretion of close-in planets of several Earth masses can probably occur only for a small fraction of protoplanetary discs (see Section 4).



**Figure 2.** Snapshots in time of the migration of a Jupiter-mass giant planet through a disc of terrestrial bodies, including the effects of aerodynamic gas drag, from a simulation by Mandell et al. (2007). Specific strong resonances with the giant planet are indicated. Colours refer to bodies' water contents, as in Fig. 1. This version reproduced from Gaidos et al. (2007).

hot Earths it is possible for the innermost hot Earth to be rocky (Alibert et al. 2006).

The main consequence of the type 1 migration model is simply that hot Earths did not form locally but farther out in the disc, probably in the water-rich icy regions. Therefore, hot Earths should contain a large quantity of ice and have measurably larger radii. In addition, the migration process favours the formation of a chain of resonant or near-resonant planets (Terquem & Papaloizou 2007).

# 3.3 Shepherding by giant planet migration

Giant planets more massive than a critical value carve an annular gap in the protoplanetary disc and are thus locked to the disc's viscous evolution (Lin & Papaloizou 1986; Takeuchi, Miyama & Lin 1996; Bryden et al. 1999; Rafikov 2002; Crida, Morbidelli & Masset 2006). These planet subsequently 'type 2' migrate, usually inward, on a  $\sim 10^5 - 10^6$  year time-scale, depending on the disc's viscosity (Lin & Papaloizou 1986; Lin, Bodenheimer & Richardson 1996; Ward 1997; D'Angelo et al. 2003). Such a planet migrates through a disc composed of both gas and solids in the form of kilometresized planetesimals and Moon- to Mars-sized planetary embryos, which formed in series of dynamical steps from micrometre-sized dust grains (as in the in situ formation model; see Section 3.1 or Chambers 2004; Papaloizou & Terquem 2006 for reviews). As the giant planet migrates inward, it shepherds material in front of strong MMRs. The evolution of a typical planetary embryo in the inner disc proceeds as follows. As the giant planet approaches the embryo, the embryo's eccentricity is increased by an MMR (usually the 2:1 or 3:2, but higher order resonances are stronger for more eccentric giant planets; Murray & Dermott 1999). Gas drag and dynamical friction with nearby planetesimals act to recircularize the embryo's orbit and decrease its energy, thereby reducing its semimajor axis and moving it just interior to the MMR (Adachi, Hayashi & Nakazawa 1976; Tanaka & Ida 1999). As the giant planet continues its migration, the embryo is again excited by the approaching MMR and the

ing giant planet migration<sup>4</sup> (Fogg & Nelson 2005; Zhou et al. 2005; Raymond, Mandell & Sigurdsson 2006b; Fogg & Nelson 2007; Mandell, Raymond & Sigurdsson 2007). However, during this process, many bodies' eccentricities are damped too slowly to avoid a close encounter with the giant planet. Such bodies are usually scattered outward, and can form a subsequent generation of exterior terrestrial planets (Raymond et al. 2006b; Mandell et al. 2007). Fig. 2 shows snapshots in time of this shepherding process from a simulation by Mandell et al. (2007). It is clear that the 2:1 MMR is responsible for the bulk of the shepherding in this simulation. The

cycle continues. Thus, embryos and planetesimals are shepherded

inward by moving MMRs and accrete into planet-sized bodies dur-

a simulation by Mandell et al. (2007). It is clear that the 2:1 MMR is responsible for the bulk of the shepherding in this simulation. The two hot Earths formed are on low-eccentricity orbits immediately interior to strong resonances, as expected. However, in simulations including weaker gas drag, hot Earths can form on higher eccentricity orbits (Fogg & Nelson 2007; Mandell et al. 2007). The survival of high-eccentricity hot Earths is uncertain, given that tides may act to alter the planets' orbits and possibly lead them into unstable giant planet resonances or drive them into the star (see Section 3.5 below).

In the giant planet migration shepherding model, hot Earths are a mixture of material that originated interior to the giant planet's orbit. Both the core accretion and gravitational collapse models predict that giant planets are likely to form at large orbital distances, beyond the snow line, which itself moves inward in time (Pollack et al. 1996; Boss 1997; Bodenheimer, Hubickyj & Lissauer 2000; Sasselov & Lecar 2000; Mayer et al. 2002). In the simulations of

<sup>&</sup>lt;sup>4</sup> The formation time-scale of shepherded hot Earths is of the order of the migration time-scale (Mandell, Raymond & Sigurdsson 2007). Thus, hot Earths may form in  $\sim 10^5 - 10^6$  yr, as opposed to the  $10^7 - 10^8$  yr time-scale for the Earth calculated from Hf/W isotopic measurements (Kleine et al. 2002; Jacobsen 2005). This very short formation time-scale for hot Earths could have consequences for their geological evolution.

Raymond et al. (2006b) and Mandell et al. (2007), the hot Earths that formed contained 1–2 per cent water by mass, as is the case for the two hot Earths in Fig. 2. The assumed starting water distribution in those cases was similar to that of present-day primitive asteroids (Abe et al. 2000; fig. 2 from Raymond et al. 2004). Note that water depletion from impacts and hydrodynamic escape was neglected in these calculations.

The giant planet migration shepherding model predicts that hot Earths should lie close to a strong MMR, most likely the 2:1 MMR, interior to a giant planet (Fogg & Nelson 2005; Zhou et al. 2005; Raymond et al. 2006b; Mandell et al. 2007). In this model, hot Earths are formed from a mixture of material that originated interior to the giant planet's orbit. Giant planets are expected to form just outside the snow line, given the increase in solid material (Hayashi 1981; Stevenson & Lunine 1988; Ida & Lin 2004). Thus, hot Earths formed by shepherding are likely to contain up to a few per cent water, but probably not more (Mandell et al. 2007). Note that planets with water contents of a few per cent by mass cannot be distinguished from rocky planets by radius measurements given observational uncertainties (Fortney et al. 2007; Seager et al. 2007; Sotin et al. 2007; Valencia et al. 2007b).

# 3.4 Shepherding by sweeping secular resonances during disc dispersal

Moving SRs can shepherd material in a similar way to MMRs if gas drag is present. An SR occurs when the apsidal precession frequency of two bodies' orbits are commensurate (e.g. Murray & Dermott 1999). In a disc with two or more giant planets, interactions between the planets cause each of their orbital alignments to precess. In addition, the gravitational potential of the massive gaseous disc affects the precession rates, and therefore the location of SRs with each planet in the disc (Ward 1981; Nagasawa, Lin & Thommes 2005). As the disc dissipates, SRs can move progressively ('sweep') across a given region, increasing the eccentricities of bodies. In the case of a smooth, inward-sweeping SR, shepherding of material can happen similar to MMR shepherding for migrating giant planets.

In the context of hot Earth formation, the SR shepherding model applies to cases with two or more giant planets that have stopped migrating. A smooth dissipation of the disc can induce SR sweeping. Much as in the migration shepherding mechanisms, a sweeping SR excites the eccentricities of nearby protoplanets. These eccentricities are subsequently damped by gas drag and the body's orbit is moved interior to the resonance. This process continues for the duration of the SR sweeping, unless a planet gets close enough to the star that its precession rate becomes dominated by general relativistic effects rather than dynamical ones (Zhou et al. 2005).

Thus, the SR shepherding model involves a complex interaction between two giant planets, the massive gaseous disc, and relatively low-mass terrestrial material. It requires a monotonic, inward SR sweeping which itself requires a smooth dispersal of the gaseous disc (Ward 1981), which is uncertain given that most stars form in large clusters and may lose disc mass in periodic photoevaporation events (Lada & Lada 2003; Adams et al. 2004; Hester et al. 2004). In the SR shepherding model, a hot Earth system must also contain at least two more distant, interacting giant planets. The compositions of hot Earths in this scenario are a mixture of material from interior to the giant planets' starting orbits. Estimating the compositions of hot Earths in this model therefore requires a knowledge of the giant planet's formation locations, specifically how far past the snow line they formed.

### 3.5 Tidal circularization of eccentric terrestrial planets

The circular orbits of hot Jupiters have been attributed to energy and angular momentum dissipation via tides raised on the planet by the star (Rasio et al. 1996). In fact, it has been proposed that close-in giant planets may have been scattered on to high-eccentricity orbits and tidally circularized (Rasio & Ford 1996; Weidenschilling & Marzari 1996; Mardling & Lin 2004; Jackson, Greenberg & Barnes 2007). Could tidal circularization act as a mechanism to transport terrestrial planets inward? To address this possibility, we integrated the second-order, coupled semimajor axis *a* and eccentricity *e* tidal evolution equations (Kaula 1964; Goldreich & Soter 1966; Greenberg 1977):

$$\frac{\mathrm{d}a}{\mathrm{d}t} = -\left(21\frac{\sqrt{GM_{\star}^{3}}R_{\mathrm{p}}^{5}k_{\mathrm{p}}}{m_{\mathrm{p}}Q_{\mathrm{p}}'}e^{2} + \frac{9}{2}\frac{\sqrt{G/M_{\star}}R_{\star}^{5}m_{\mathrm{p}}k_{\star}}{Q_{\star}'}\right)a^{-11/2},\quad(1)$$

$$\frac{\mathrm{d}e}{\mathrm{d}t} = -\left(\frac{21}{2}\frac{\sqrt{GM_{\star}^{3}}R_{\mathrm{p}}^{5}k_{\mathrm{p}}}{m_{\mathrm{p}}Q_{\mathrm{p}}'} + \frac{171}{16}\frac{\sqrt{G/M_{\star}}R_{\star}^{5}m_{\mathrm{p}}k_{\star}}{Q_{\star}'}\right)a^{-13/2}e,\,(2)$$

where  $Q'_p$  and  $Q'_{\star}$  are the tidal dissipation functions of the planet and star, respectively,  $k_p$  and  $k_{\star}$  are the Love numbers of the planet and star,  $m_p$  and  $M_{\star}$  are the masses of the planet and star,  $R_p$  and  $R_{\star}$  are the radii of the planet and star, and G is the gravitational constant.

Note that solutions with higher order terms in e have been derived (e.g. Hut 1981; Eggleton, Kiseleva & Hut 1998). However, such models include, in effect, assumptions about how a body responds to the ever-changing tidal potential, effects that have not been observed. Therefore not enough is known about the actual response of real bodies to evaluate these higher order effects. As we are only interested in the qualitative differences between planets whose orbits have evolved through tidal decay and those that did not, the second-order solution should suffice.

We considered a stellar mass of 0.3, 1 and  $3 M_{\odot}$  with radii determined from Gorda & Svechnikov (1999), a planet mass of 1 and  $5 M_{\oplus}$  (assuming  $R_p \propto m_p^{0.27}$ , as suggested by Valencia et al. 2006), a perihelion distance from 0.025 to 0.1 au, and eccentricities *e* from 0 to 0.9. For the planet, we assumed  $k_p = 0.3$  and  $Q'_p = 21.5$  (Dickey et al. 1994; Mardling & Lin 2004); for the star,  $k_* = 1.5$  and  $Q'_* = 10^{5.5}$  (Jackson et al. 2007; Ogilvie & Lin 2007). Each orbit was integrated for 10 Gyr using a  $10^3$  year time-step, which convergence tests showed is three orders of magnitude smaller than necessary to produce reliable results.

Fig. 3 shows the evolution of a set of  $5 M_{\oplus}$  planets with the same initial perihelion distance of 0.025 au, and starting eccentricities ranging from 0.05 (a = 0.026 au) to 0.9 (a = 0.25 au). As expected, evolution proceeds much faster for bodies at smaller orbital distances (in this case, those with lower eccentricities). For planets with starting eccentricities of 0.6 or less ( $a \leq 0.06$  au), orbital circularization takes place within 108 yr, including an inward drift in semimajor axis of up to 0.01-0.02 au. Circularization takes longer for larger a values, but the amount of inward drift is also increased. For the e = 0.8 planet, circularization requires several Gyr, but the planet moves inward from 0.125 to 0.065 au. At still-larger orbital distances (and eccentricities), circularization takes longer than the age of the star. Note that orbital evolution continues slowly after the planet's orbit becomes circular, via tides raised on the star by the planet. Mardling & Lin (2004) showed that should a become very small ( $\leq 0.01$  au), then the planet is doomed to fall into the star within a few Gyr. We confirm that assessment here. Therefore we expect no planets inside 0.01 au for any formation scenario.



**Figure 3.** Orbital semimajor axis *a* and eccentricity *e* versus time for a series of 5  $M_{\oplus}$  planets orbiting Sun-like stars with starting perihelion distances of 0.025 au. Each curve corresponds to a planet with a given starting eccentricity, from 0.05 (*a* = 0.026 au) to 0.9 (*a* = 0.25 au). If *a* drops below ~0.01 au, then the planet will fall into the star due to tidal evolution.

The degree to which tidal circularization can move a planet inward clearly depends on its starting orbit, size and ability to dissipate energy. Fig. 3 shows the most tidal evolution of any of the cases we explored for a solar-mass star; in most cases evolution was slower (except for the  $3 M_{\odot}$  cases, which were faster). Thus, it appears that inward movement of planets during tidal evolution is relatively small, at most a ~0.05 au change in semimajor axis. However, if the large eccentricity were due to a perturbative event, then the planet's starting aphelion might be representative of its pre-encounter semimajor axis. In that case, the effective inward movement due to tides is doubled (starting aphelion to final semimajor axis), although it would still be less than about 0.1–0.15 au on a >Gyr time-scale. The composition of the planet depends on its formation history, especially whether it formed locally or migrated inward.

What is the source of the large eccentricity needed to drive tidal circularization? Planet–planet scattering has been invoked to explain the large eccentricities of the known extrasolar planets (Rasio et al. 1996; Weidenschilling & Marzari 1996; Lin & Ida 1997; Ford, Lystad & Rasio 2005; but see Barnes & Greenberg 2007). The strength of a scattering event depends on a combination of the escape speed of the perturber, the encounter velocity and the escape speed from the system. For close-in planets, the system escape speed is large, and so only very massive bodies can excite large eccentricities. Indeed, accretion may be preferred over scattering in these situations (Goldreich, Lithwick & Sari 2004).

One alternative mechanism for eccentricity growth is an instability in nearby giant planets could alter the orbit of a terrestrial planet (Veras & Armitage 2006). In that case, one or more exterior giant planets should exist in the system, on eccentric orbits. Another possible source of eccentricity could arise if the host star had a binary companion. If the orbital plane of the planet were significantly inclined with respect to that of the binary, then large eccentricities could be induced via the Kozai mechanism (Kozai 1962). In most of these models, some evidence for an external perturber should be evident.

Thus, tidal circularization can move a highly eccentric terrestrial planet inward to some extent, although the planet must be relatively massive  $(\gtrsim 5 M_{\oplus})$  and have a very small starting perihelion distance

 $(\lesssim 0.03 \text{ au})$ . If the planet formed locally, then its composition is likely to be relatively dry ( $\lesssim 1$  per cent water by mass). A source of high eccentricity may also be evident, such as a binary stellar companion or a distant eccentric giant planet.

# 3.6 Giant planet migration and photoevaporation

Baraffe et al. (2004, 2006) proposed that close-in, Neptune-mass planets might be the remains of larger planets that have been photoevaporated away. Such planets would form farther from their parent stars and migrate inward (Ida & Lin 2004; Alibert et al. 2005), losing a portion of their gaseous envelopes hydrodynamically via irradiative XUV heating (Lammer et al. 2003; Baraffe et al. 2004). Here we investigate the possibility that photoevaporation could lead to the removal of the entire envelope of a hot Jupiter or hot Neptune, leaving behind a solid planet, i.e. the core of the irradiated giant planet.

Recent estimates derive evaporation rates that are far smaller than those used by Lammer et al. (2003), and include the effects of twodimensional layering (Tian et al. 2005) and improved atmospheric chemistry (Yelle 2004, 2006). Indeed, these new evaporation rates are closer to those of Watson, Donahue & Walker (1981). Perhaps most convincing that the Lammer et al. evaporation rates are too large is empirical evidence that the mass distribution of highly irradiated extrasolar planets (inside 0.07 au) is identical to that of more distant planets (Hubbard et al. 2007a). A substantial change in the mass function is predicted for evaporation models (i.e. fewer massive planets and more less massive ones).<sup>5</sup> Such an effect may exist at lower masses, but not in the currently probed sample of planets. Hubbard et al. (2007b) show that at the minimum orbital radius of known extrasolar planets (0.023 au), the initial mass must be less than about a Saturn mass to evaporate completely, i.e. to its core. For more typical hot Jupiter orbits, at 0.05–0.1 au, this critical mass is smaller still.<sup>6</sup> The models of Baraffe et al. (2004) and Hubbard et al. (2007b) do not account for the presence of a core, which is important once the planet mass is less than  $\sim 100 \,\mathrm{M}_{\oplus}$ , such that a 5–10  $M_{\bigoplus}$  core constitutes a non-negligible fraction of the planet mass. Note that the Baraffe et al. (2006) models do incorporate this effect, as we do implicitly by using their internal structure models.

Mass-loss due to hydrodynamic escape, limited only by energy deposition, depends critically on the stellar irradiance of the atmosphere, and can be approximated by the relation

$$\dot{M} = \frac{3\,\beta(F_*,a)^3}{G\rho}\,\frac{F_{\rm XUV} + F_\alpha}{a^2},\tag{3}$$

where  $F_{XUV}$  and  $F_{\alpha}$  represent the high-energy radiation incident on the planet, and  $\rho$  and a are the planet density and orbital distance from the star, respectively (Lammer et al. 2003; Baraffe et al. 2004).

<sup>5</sup> Fortney et al. (2007) also showed that if hot Neptunes form via photoevaporation of hot Jupiters, then their radii should be of the order of one Jupiter radius. However, if they are not remnants of hot Jupiters, then their radii should be  $0.3-0.4R_J$ . The first transiting hot Neptune indeed has a radius of ~0.35-0.4  $R_J$  (Deming et al. 2007; Gillon et al. 2007). Note, however, that Gliese 436 is an M dwarf (0.41 M<sub>☉</sub>) and therefore has low EUV and FUV emission (except during flares), which are key for driving evaporative mass-loss (Butler et al. 2004).

<sup>6</sup> Tidal evolution models suggest that close-in planets may have originated at somewhat larger orbital distances and slowly evolved inward, concurrently decreasing their semimajor axes and eccentricities (Jackson et al. 2007). If close-in giant planets did indeed originate on somewhat more distant, eccentric orbits, then their time-averaged fluxes would likely be reduced.

The parameter  $\beta$  is the ratio of the irradiated planetary radius to the planet's 'original', non-irradiated radius for a specific stellar flux and orbital distance (Lammer et al. 2003; Baraffe et al. 2004, 2006; Hubbard et al. 2007b all assume  $\beta = 3$  based on atmospheric models of Watson et al. 1981). For a constant orbital distance, the mass-loss will therefore initially decrease in time as the planet cools and becomes more dense and the star's UV and X-ray flux decreases. Over time, as hot material escapes from the top of the planetary atmosphere new layers are irradiated and stellar flux is converted to expansion energy, gravitational contraction of the planet slows. If enough mass is evaporated, expansion surpasses contraction and the planet experiences runaway mass-loss, leaving behind only the solid core.

To construct a simplified model of photoevaporative mass-loss, we need to constrain certain parameters. We assumed evaporation rates 100 times smaller than the energy-limited case from Lammer et al. (2003). The radius of an evaporating planet stays relatively constant regardless of mass, such that we extrapolated radii for planets of various masses and heavy-element abundances from Baraffe et al. (2006, with corrections for semimajor axis from Chabrier et al. 2004) to find a mass-radius relation for irradiated planets as a function of time. Planets less massive than  ${\sim}50\,\mathrm{M}_{igodot}$  are more likely than larger planets to contain significant concentrations of molecular species, simply because the ratio of core mass to envelope mass is decreased (e.g. Uranus and Neptune). Molecules such as H<sub>2</sub>O and CH<sub>4</sub> play an important part in the energetics of atmospheric expansion, and therefore affect the mass-loss rate (Hubbard et al. 2007a). Although our approach does not directly incorporate changes in evaporation rate with chemistry, the mass-radius relations from Baraffe et al. (2006) are based on the values of Alibert et al. (2005) for heavyelement enrichment of the planets' atmospheres and therefore implicitly include a decreased evaporation rate for smaller planets with heavy-element-rich atmospheres since evaporation rates depend on the planet radius. Our simplified model demonstrates good agreement with the results from more detailed models by Hubbard et al. (2007b) and the reduced-evaporation models of Baraffe et al. (2006) for higher mass planets.

Fig. 4 shows the masses of two highly irradiated planets as a function of time, on circular orbits at 0.025, 0.05 and 0.1 au. Even on its closest orbit, the more massive planet  $(72 \, M_{\oplus})$  took 5 Gyr to evaporate to its core. Given that the transition from type 1 to type 2 migration is thought to occur at roughly  $70 \, M_{\oplus}$  (Ward 1997; D'Angelo et al. 2003), this effectively rules out the formation of hot Earths by type 2 migration and subsequent photoevaporation, simply because planets massive enough to type 2 migrate will not lose enough mass by photoevaporation. In contrast, the less massive planet ( $25 \, M_{\oplus}$ ) evaporated to its core in less than 50 Myr at 0.025 au, but required Gyr to lose mass past 0.05 au. Additionally, beyond ~0.1 au the mass-loss over 5 Gyr is negligible; planets found beyond this orbital distance maintain their mass over the lifetime of the system.

After the primordial atmosphere has been lost, the core of the planet is exposed. If the core of the planet is composed primarily of low-temperature condensates (consistent with formation in the cold outer disc – Pollack et al. 1996; Boss 1997), the outer layers of the planetary core may continue to vaporize. To determine the original mass of a hot Earth formed from the core of a photoevaporated massive planet, models of the evolution of intensely irradiated icy bodies must be developed; current models of volatile-dominated Earth-mass planets (Valencia et al. 2006; Fortney et al. 2007; Seager et al. 2007; Sotin et al. 2007; Valencia et al. 2007a,b) have not yet probed these processes.



**Figure 4.** The evolution of the masses of two highly irradiated planets due to photoevaporation. Our model is based on planetary structure models of irradiated planets from Baraffe et al. (2006) and Chabrier et al. (2004) and evaporation rates based on the model of energy-limited hydrodynamic escape from Lammer et al. (2003), but reduced by a factor of 100 in accordance with the results of Hubbard et al. (2007a). The intermediate-mass planet (72 M<sub> $\oplus$ </sub>) represents the boundary between planets that would undergo type I migration ( $M \lesssim 70 M_{\oplus}$ ) and planets that would undergo type II migration ( $M \gtrsim 70 M_{\oplus}$ ; D'Angelo et al. 2003). The planets are placed at three different orbital radii: 0.025 au (solid), 0.05 au (dotted) and 0.1 au (dashed).

Thus, photoevaporation of close-in gaseous planets may remove their atmospheres and leave behind solid cores. This process is only effective for planets within about 0.05 au that are below the type 2 migration threshold of  ${\sim}70\,M_{\oplus}$ . For those cases, 'hot Neptunes' could become 'hot Earths' on a 108-1010 yr time-scale. This process can occur in conjunction with the type 1 migration scenario discussed above in Section 3.2. Indeed, the most likely source of hot Neptunes is the outer disc (Gillon et al. 2007). In some cases, a series of  $\gtrsim 10 \,\mathrm{M}_{\oplus}$  planets may form in the cold outer disc and then type 1 migrate inward as described above, into a chain of hot Neptunes. Depending on their orbits, the innermost planet or two could be photoevaporated over time into a very water-rich hot Earth. A diagnostic of photoevaporation could therefore be a system with (1) a water-rich hot Earth inside 0.05 au and (2) additional,  $\sim$  Neptune-mass planets exterior to the hot Earth in near-resonant orbits. However, it would still be difficult to definitely assess the degree of photoevaporation in such a setting.

More definitive detections of the importance of photoevaporation would require statistics of a large number of hot Earths and hot Neptunes orbiting stars with a range of ages. A correlation between the number of hot Earths versus hot Neptunes and the stellar age would indicate that such planets were losing mass in time, presumably via photoevaporation. Alternatively, exploring the mass functions of close-in planets down to lower masses could reveal time-dependent mass-loss (as in Hubbard et al. 2007a).

# 4 ORIGIN OF THE KNOWN HOT EARTH SYSTEMS

# 4.1 Gliese 876

Gliese 876 is a  $0.32 \,M_{\odot}$  star (M4 dwarf) less than 5 pc from the Sun (Marcy et al. 1998). Its known planetary system contains a  $\sim 7.5 \,M_{\oplus}$  hot Earth at 0.02 au, as well as two additional, around Jupiter-mass

planets in a 2:1 resonance on more distant orbits (Marcy et al. 1998, 2001; Rivera et al. 2005). The separation between the hot Earth and the giant planets is significant: the ratio of orbital periods between the hot Earth and inner giant planet is 16.6. Both models and observations suggest that Jovian planets are rare around low-mass stars (Laughlin, Steinacker & Adams 2004; Ida & Lin 2005; Butler et al. 2006; Endl et al. 2006; Gould et al. 2006). Thus, the existence of two such massive planets around GJ 876 may indicate that its protoplanetary disc was particularly massive (e.g. Lovis & Mayor 2007; Wyatt, Clarke & Greaves 2007).

Could the GJ 876 hot Earth at 0.02 au have formed in situ? If so, then there must have been at least  $7.5 M_{\oplus}$  in solids interior to  $\sim 0.05$  au, assuming accretion was efficient. In the MMSN model, assuming the surface density  $\Sigma$  scales as  $r^{-3/2}$ , there is  $\sim 0.75 \,\mathrm{M}_{\oplus}$ inside 0.05 au, assuming the disc to extend all the way into the star (Weidenschilling 1977; Hayashi 1981; Raymond et al. 2007). For a more common disc profile of  $\Sigma \propto r^{-1}$  (e.g. Andrews & Williams 2007), and applying an MMSN prescription, there is only  $0.06 \,\mathrm{M}_{\oplus}$ inside 0.05 au. Thus, if 7.5  $M_{\oplus}$  of material existed in the inner 0.05 au of GJ 876's disc, then that disc must have been 10-100 times more massive than the solar nebula. This value is rather large, but it is not outside the realm of possibility, given the large spread in observed disc masses (Andrews & Williams 2005; Scholz et al. 2006). However, such a massive disc would be an anomaly, and the fraction of discs that could form such a close-in planet is small (Raymond et al. 2007). In addition, given the  $\sim$  linear relation between disc mass and stellar mass (e.g. Scholz et al. 2006), such a massive disc is an additional three times less likely. In addition, given the large dynamical separation between the hot Earth and the closest Jovian planet, there is no clear explanation for the lack of additional hot Earths. Thus, it is unlikely that GJ 876's hot Earth formed in situ.

If the GJ 876 hot Earth formed at a distance and type 1 migrated inward, we would expect it to have companions of similar mass in near resonant orbits. No such companions have been discovered to date, although planets of a few  $M_{\oplus}$  would probably not be detectable (Rivera et al. 2005). However, given the existence of the two Jovian planets, perhaps there was a limited window of time for type 1 migration into the inner disc: once the giant planets formed, they would pose a barrier for smaller migrating bodies (Thommes 2005). A mass of 7.5  $M_{\oplus}$  is consistent with a single hot Earth migrating into the inner disc, then stalling where the type 1 torques disappear (Masset et al. 2006b; Paardekooper & Mellema 2006).

Zhou et al. (2005) explain the origin of the GJ 876 hot Earth with a combination of shepherding from giant planet migration and SR sweeping. In the model of Zhou et al. (2005), the two Jovian planets formed on more distant orbits, and were trapped in resonance during migration (e.g. Lee & Peale 2002). This migration also induced the formation of planets inside strong resonances, by the migration shepherding mechanism described in Section 3.3. The giants' migration stalled close to their current orbits, but subsequent dissipation of the disc induced SR sweeping, promoting further accretion and shepherding the hot Earth farther away from the giant planets. This two-step model does not require as large a disc mass as in situ accretion, because some mass from more distant regions is shepherded into the inner disc. In addition, it predicts a significant separation between the giant planets and the hot Earth, caused by the SR sweeping after migration. However, this model has some uncertainties. For example, the violent nature of star-forming environments may cause episodic pulses in the evaporation of the disc (Adams et al. 2004) and therefore in the location of SRs (Ward 1981). In such non-monotonic SR sweeping, it is unclear if material can still be shepherded.

In the model of Zhou et al. (2005), it is likely that the hot Earth is relatively dry, assuming its composition is determined by the formation zone of the innermost giant planet, and that accretion followed roughly as in Mandell et al. (2007). In the type 1 model, the hot Earth could be rocky or icy, also depending on its formation zone. If GJ 876 d were to transit its host star, then its bulk composition (rocky versus icy) could be determined (see Section 2). If it were shown to be icy in nature, that would support the type 1 migration scenario. However, if it were rocky, it would lend support to the model of Zhou et al. (2005).

### 4.2 Gliese 581

Gliese 581 is a  $0.31 \,\mathrm{M}_{\odot}$  M3 dwarf at a distance of 6.3 pc from the Sun (Hawley, Gizis & Reid 1997). Its planetary system contains three hot Earths/Neptunes with orbits between 0.04 and 0.25 au and minimum masses between 5 and  $15 \,\mathrm{M}_{\oplus}$  (Bonfils et al. 2005; Udry et al. 2007). The innermost planet is the hot Neptune ( $M \sin i = 15.7 \,\mathrm{M}_{\oplus}$ ). No Jovian planets have been detected in the system to date, ruling out the two shepherding mechanisms.

The most likely formation mechanism of the GJ 581 system is either *in situ* formation or type 1 migration (Raymond et al., in preparation). The orbital periods of the planets do not form an obvious pattern – the period ratios between planets b/c and c/d are 2.38 and 6.34, respectively. The semimajor axes of planets b/c and c/d are separated by 20.5 and 47 mutual Hill radii, respectively, similar to values for the Solar system's terrestrial planets.

For the GJ 581 planets to have formed *in situ* would require  $\sim$ 40–50 M $_{\oplus}$  inside 0.5 au. Indeed, the simulation from Fig. 1 is an attempt to reproduce the system via *in situ* accretion. By the same arguments as made above, this would require a disc that is, at least in its inner regions, 17–50 times more massive than a minimum-mass disc. Given that the spacing of planets b, c and d is comparable to those of Venus, Earth and Mars, *in situ* accretion remains a reasonable model for GJ 581. In this scenario, the innermost planet would have accreted first, and therefore may have been able to capture a small amount of nebular gas to account for its large mass (e.g. Pollack et al. 1996).

Formation at larger orbital distances followed by type 1 migration is the other viable mechanism for GJ 581. The planets' spacings are not next to obvious resonances, but b/c lie less than 10 per cent from the 5:2 MMR and quite close to the 12:5. Planets c/d are more distant, but of course there exists the possibility of an additional, slightly lower mass planet between planets c and d. If such a planet were discovered, it would support the type 1 migration scenario.

Tidal effects are important in the GJ 581 system, given the planets' proximity to the star. Given that tides damp both semimajor axes and eccentricities, it is likely that the GJ 581 planets b and c formed on more distant and more eccentric orbits (Barnes et al. 2007). Given the planets' already significant eccentricities ( $e \sim 0.2$  for each planet), it is not clear how the system could form with such high eccentricities. In addition, the fact that the innermost planet is the most massive of the three suggests that photoevaporation has not occurred in this system. Indeed, given the star's low luminosity (1.3 per cent of solar), the threshold distance for photoevaporation is likely to be at less than 0.01 au.

Despite the uncertainties, the main difference between the two possible models is simply the composition of hot Earths. *In situ* formation predicts relatively dry planets, while type 1 migration predicts icy planets with >10 per cent water by mass. Thus, if transits were measured for any of the GJ 581 planets and a composition were

determined, then it would be possible to distinguish between these two models.

# **5 SUMMARY AND DISCUSSION**

We anticipate that a large number of planetary systems containing close-in terrestrial planets, referred to here as 'hot Earths', will be discovered in the coming years with RV and transit measurements. In some cases, both an accurate determination of the architecture of the inner planetary system and of the bulk composition of a hot Earth (rocky versus icy; but see Adams et al. 2007) will be possible (see Section 2). The goal of this paper is to determine whether the formation history of such systems can be unravelled, given the relatively small amount of information available. In addition to four alreadyknown mechanisms for hot Earth formation, we have shown that tidal circularization of highly eccentric planets can move terrestrial planets' orbits inward, but only by perhaps 0.1 au, and only for very close-in perihelion distances ( $\leq 0.05$  au). In addition, our simple model suggests that photoevaporation can remove a giant planet's atmosphere and leave behind the core. However, this is only possible for very close-in orbits (<0.025–0.05 au) and relatively low-mass planets ('hot Neptunes' with masses below  $70 M_{\oplus}$ ), as suggested by Hubbard et al. (2007a).

Table 1 summarizes the observable consequences of these six models for hot Earth formation. There exist several clear differences between the models that should be detectable in the near future. Given a planetary system with a transiting hot Earth, considerable RV measurements, and perhaps transit timing analysis, Table 1 provides a simple way to determine the formation history of hot Earth planetary system. Note that in some cases, more than one of these mechanisms can act in concert. For example, the case of GJ 876 may be explained in a two-step process, via shepherding during migration and then during SR sweeping (see Section 4.1; Zhou et al. 2005). In addition, tides affect the orbits of all hot Earths to some degree, regardless of their formation history. However, certain mechanisms cannot act together: planets massive enough to type 2 migrate cannot have their envelopes photoevaporated and become hot Earths (see Section 3.6).

The formation mechanisms of the two known hot Earth systems are not entirely clear (see Section 4 above). However, transit measurements of the hot Earth of either of the known systems would make it far easier to discern between models. In particular, for the case of GJ 581, a transit measurement of planet c or d would distinguish between *in situ* formation (rocky) and type 1 migration (icy). Clearly, more work is needed to better characterize and quantify some of these models, and to examine the long-term survival of hot Earths in different systems. In addition, it is possible that additional mechanisms exist for hot Earth formation that have not yet been considered.

Zhou et al. (2005) claimed that hot Earths should be numerous if giant planets form via core accretion (Mizuno 1980; Pollack et al. 1996; Lissauer & Stevenson 2007), but rare if they form via gravitational instability (Boss 1997; Mayer et al. 2002; Durisen et al. 2007). Given the large number of avenues for hot Earth formation, we disagree with Zhou et al. (2005) on this point. Indeed, three of the candidate mechanisms for hot Earth formation – *in situ* accretion, type 1 migration, and tidal circularization – do not require a giant planet at all and so are unaffected. Photoevaporation of hot Neptunes may be affected because the two giant planet formation models predict different core masses: core accretion predicts 5–20 M $\oplus$  cores (e.g. Alibert et al. 2005) while the cores of giant planets formed via disc instability are likely to be smaller (Boss 1998, 2006). For the

other two mechanisms - giant planet migration shepherding and SR shepherding - is there a reason that the outcome should depend on the mode of giant planet formation? The main difference between the two models is the timing of giant planet formation: core accretion predicts that giant planets form late in the lifetime of the gaseous disc, while gravitational instability forms planets very quickly. Giant planet migration starts immediately after, or even during, formation (Lufkin et al. 2004). Thus, if giant planets form via core accretion, they migrate through a disc that has undergone at least 1 Myr of accretion, and contains both around Moon-sized planetary embryos and kilometre-sized planetesimals (plus ~99 per cent gas; e.g. Kokubo & Ida 2000; Chambers 2004). If, however, giant planets form via gravitational instability, then they would migrate through a disc containing predominantly smaller bodies such as planetesimals. Fogg & Nelson (2005, 2007) showed that the prevalence of shepherding versus scattering during migration is relatively insensitive to the accretion history of the inner disc. In the SR shepherding model, two giant planets must be on interacting orbits by the late stages of the dispersal of the gaseous disc; the planets' prior orbital histories are not relevant. Thus, we see no reason that the abundance or rarity of hot Earths should be affected by the mechanism for giant planet formation.

One other interesting difference between the core accretion and gravitational instability models is the expected location of giant planet formation. In core accretion, there are several reasons to expect giant planets to form just past the snow line: (1) the density of solid building blocks increases by a factor of 2-4 or more (Hayashi 1981; Stevenson & Lunine 1988; Lodders 2003), (2) accretion timescales are shorter than anywhere else beyond the snow line (Kokubo & Ida 2002; Ida & Lin 2004) and (3) the surface density jump at the snow line, if it is steep enough, can trap inward-migrating planetary cores and form a pileup (Masset et al. 2006b). If these arguments hold, then core accretion predicts that material interior to the giant planet is therefore relatively dry. However, gravitational instability forms planets in the more distant reaches of protoplanetary discs, where the Toomre Q value is lowest (Boss 1997; Mayer et al. 2002). Thus, icy material is included interior to the giant planet. In the giant planet migration shepherding model, hot Earths are a mixture of material interior to the giant planet's starting orbit (Mandell et al. 2007); they would be rocky for the core accretion model, and icy for the instability model. Thus, transit measurements of hot Earth in systems formed by giant planet migration shepherding may provide a test to distinguish between the two dominant giant planet formation models.

As observational uncertainties of planetary orbits and masses become smaller, it will become possible to differentiate formation mechanisms based on these observations. We have laid out the qualitative differences between six different mechanisms that may form hot Earths (although some phenomena may operate simultaneously). Determining how hot Earths form is an important step toward understanding planet formation, identifying target stars for future surveys, and searching for habitable planets.

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