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# PREDICTING PLANETS IN KNOWN EXTRASOLAR PLANETARY SYSTEMS. I. TEST PARTICLE SIMULATIONS

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## ABSTRACT

Recent work has suggested that many planetary systems lie near instability. If all systems are near instability, then at least one additional planet must exist in stable regions of well-separated extrasolar planetary systems to push these systems to the edge of stability. We examine the known systems by placing massless test particles in between the planets and integrating for 1–10 million yr. We find that some systems, HD 168443 and HD 74156, eject nearly all test particles within 2 million yr. However, we find that HD 37124, HD 38529, and 55 Cnc have large contiguous regions in which particles survive for 10 million yr. These three systems, therefore, seem the most likely candidates for additional companions. Furthermore, HD 74156 and HD 168443 must be complete; therefore radial velocity surveys should only focus on detecting more distant companions. We also find that several systems show stable regions that only exist at nonzero eccentricities.

*Subject headings:* celestial mechanics — methods: numerical — planetary systems

## 1. INTRODUCTION

Several categories of planetary systems have been discovered in the past several years. Some appear dynamically similar to our own solar system (SS), but others appear quite different (Barnes & Quinn 2004, hereafter BQ04). Whether these systems really fall into unique categories or are instead the first examples of a continuous spectrum of stability remains to be seen. Nonetheless, we will define three categories of systems: resonant, interacting, and separated. For the purposes of this paper, we will define a planetary system as a star with two or more companions: at least a three-body system. Resonant systems contain two or more planets in mean-motion resonances. The GJ 876 (Marcy et al. 2001b) and HD 82943[2] systems are in the 2:1 resonance, and the two inner planets of the 55 Cnc system (Marcy et al. 2002) are in 3:1 resonance. The interacting systems of $\upsilon$ And (Butler et al. 1999), HD 12661 (Fischer et al. 2003), 47 UMa (Fischer et al. 2002), and the SS are not in resonance, but their orbits are close enough that the planets may perturb each other. The final category is the separated systems. In these systems, the planets are separated enough that they are not interacting on long timescales ($\lesssim 10^6$ yr). This paper will focus on these separated systems, specifically HD 168443 (Marcy et al. 2001a), HD 74156,[3] HD 38529 (Fischer et al. 2003), and HD 37124 (Butler et al. 2003). We will also examine 55 Cnc, which currently appears to be a combination of a resonant and a separated system.

Studies of the known resonant and interacting systems have demonstrated that large regions of complete instability lie in observationally permitted parameter space (BQ04). Slight changes in orbital elements, specifically eccentricity or proximity to perfect resonance, lead to a catastrophic disruption of the systems. This suggests that many systems lie on the verge of instability. Planetary systems near instability are as tightly packed as possible; there is no room for additional companions. In this paper we explore the possibility that all systems are tightly packed, implying that additional companions lie between the extant planets in separated systems. In this "packed planetary systems" (PPS) hypothesis, the undetected planets have not been observed because there are not enough data to discover the additional planet, or the planetary mass falls at or below the detection limit of current Doppler technology. BQ04 also examined the stability of the Sun-Jupiter-Saturn system and showed that it lies further from instability than the complete gas giant system. The Sun-Jupiter-Saturn model is the only system known to be incomplete and therefore supports the hypothesis that all systems lie near instability and are hence packed.

The PPS model assumes that planet formation is an efficient process. As many planets (or at least gas giants) form in circumstellar disks as possible. This is seen in the gas giant region of the SS, as well as in the resonant and coupled systems. In this scenario, as dust congeals into ever larger bodies, the protoplanets perturb each other in ever increasing amounts as the masses grow. As the protoplanets become larger, they acquire the ability to pull other planets into favorable positions (dynamically stable) or, if the perturbations become strong enough, eject them completely from the system. The implication is that any stable region in between known planets harbors an (as yet) unseen companion. In this paper we attempt to map out stable regions in semimajor-axis and eccentricity space of known separated systems.

We perform numerical simulations using test (massless) particles to search for stable regions in HD 168443, HD 74156, HD 37124, HD 38529, and 55 Cnc. In addition to these systems, we also perform a control experiment. We replace the middle planet of the $\upsilon$ And system with a belt of test particles to evaluate how well this experiment predicts the orbit of $\upsilon$ And c. We find that some systems, HD 74156 and HD 168443, contain no stable regions and are complete to the most distant planet. HD 37124, HD 38529, and the resonant/separated system 55 Cnc each contain a zone of stability that may harbor an undetected planet. This paper is divided into the following sections. In § 2 we describe the numerical techniques used to examine these systems. In § 3 we present the results for systems that may have additional companions. In § 4 we present

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[2] See http://obswww.unige.ch/~udry/hd82943syst.html.

[3] See http://obswww.unige.ch/~udry/planet/hd74156.html.

TABLE 1

Orbital Parameters of Selected Planetary Systems

| System | Planet | $M$ ($M_J$) | $P$ (days) | $a$ (AU) | $e$ | $\varpi$ | $T$ (JD) |
|---|---|---|---|---|---|---|---|
| HD 168443 | b | 7.73 | 58.1 | 0.295 | 0.53 | 172.9 | 2,450,047.58 |
| | c | 17.2 | 1770.0 | 2.87 | 0.2 | 63 | 2,450,250.6 |
| HD 74156 | b | 1.56 | 51.610 | 0.28 | 0.649 | 183.7 | 2,451,981.40 |
| | c | 7.5 | 2300.0 | 3.47 | 0.395 | 240.0 | 2,450,849.0 |
| HD 37124 | b | 0.86 | 153.0 | 0.54 | 0.1 | 97.0 | 2,451,227 |
| | c | 1.01 | 1942 | 2.95 | 0.4 | 265.0 | 2,451,828 |
| HD 38529 | b | 0.78 | 14.309 | 0.129 | 0.29 | 87.7 | 2,450,005.8 |
| | c | 12.8 | 2174.3 | 3.68 | 0.36 | 14.7 | 2,450,073.8 |
| 55 Cnc | b | 0.84 | 14.653 | 0.115 | 0.02 | 99.0 | 2,450,001.479 |
| | c | 0.21 | 44.276 | 0.241 | 0.339 | 61.0 | 2,450,031.4 |
| | d | 4.05 | 5360 | 5.9 | 0.16 | 201.0 | 2,452,785 |
| $\upsilon$ And | b | 0.69 | 4.61706 | 0.059 | 0.015 | 32.0 | 2,450,001.588 |
| | c[a] | 1.96 | 241.1 | 0.83 | 0.25 | 251.0 | 2,450,160.1 |
| | d | 3.98 | 1309 | 2.5 | 0.34 | 255.0 | 2,450,044.0 |

[a] Not used in these simulations.

the results of the $\upsilon$ And system, with the middle planet removed. We summarize our results in § 5 and suggest future work in this field. This is the first in a series of papers that will explore the possibility of additional planets in known systems. In Paper II we will place massive planets in the stable regions identified by this work to refine our predictions. In Paper III we will examine how terrestrial-planet formation might occur between the detected planets.

## 2. NUMERICAL METHODS

These simulations were performed with SWIFT[4] (Levison & Duncan 1994). Specifically, we used the regularized mixed variable symplectic integrator, RMVS3. This code is designed to quickly integrate a system through close approaches (via regularization), yet still maintain conservation of energy (via a symplectic algorithm). For most systems (except 55 Cnc and $\upsilon$ And), we integrate for $10^7$ yr. We require every simulation to conserve energy to a factor of $10^{-4}$, which has been shown to be accurate enough for simulations of planetary system stability on these timescales (BQ04). Generally we conserve energy to better than 1 part in $10^6$.

The systems presented here are all coplanar. The mass-inclination degeneracy of Doppler observations is broken by assigning the companions' masses to be the observed minimum masses. All other orbital elements are their best-fit values. Note that in coplanar systems, the longitude of ascending node, $\Omega$, is meaningless. Test particles are all also coplanar. They are spaced every 0.002 AU in semimajor-axis space, and every 0.05 in eccentricity space. Their mean anomalies are placed uniformly throughout $[0, 2\pi)$. In noncircular trials the longitudes of periastron, $\varpi$, of the test particles were aligned with the most massive planet. This alignment is arbitrary, but probably valid for low $e$ ($<0.1$). However, some research suggests that for $e > 0.1$ the apses should have been antialigned (Laughlin et al. 2002), although this alignment is not strictly necessary (BQ04).

This procedure is similar to other work on test particles in $\upsilon$ And (Rivera & Lissauer 2000), GJ 876 (Rivera & Lissauer 2001), and 55 Cnc (Rivera & Haghighipour 2003), the major difference being that we examine noncircular test particle orbits. In addition, Menou & Tabachnik (2003) placed test particles in the habitable zones of these systems. With the exception of HD 37124, we recover their results.

## 3. RESULTS

In Table 1 the orbital elements of all the planetary systems used in this paper are shown. Since there are no published data for some systems, we are forced to use data from the Internet, which are constantly changing. In § 3.1 we examine the

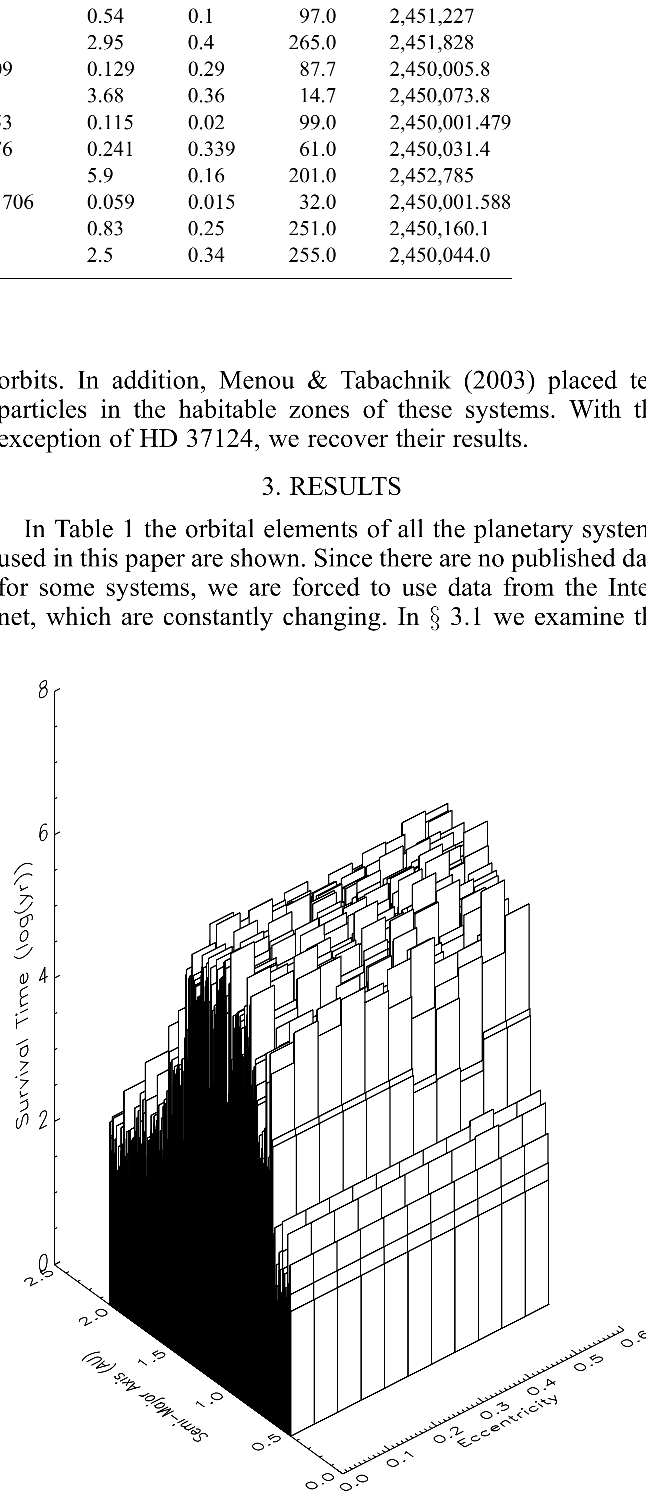

Fig. 1.—Stability of test particles in HD 168443. The height of the bars corresponds to how long test particles remained bound to the parent star. Regardless of eccentricity, no test particles can survive in this system for even 2 million yr.

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[4] SWIFT is publicly available at http://www.boulder.swri.edu/~hal/swift.html.

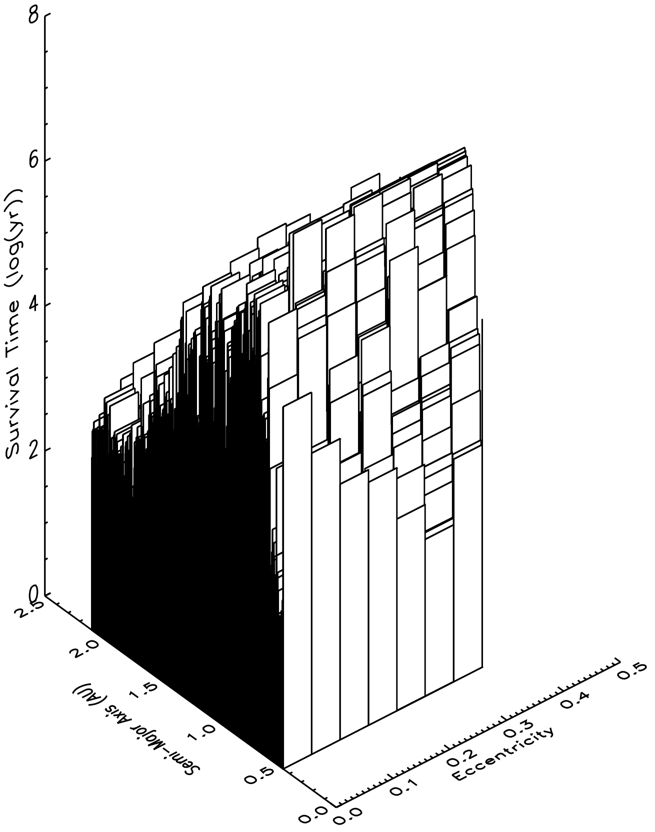

FIG. 2.—Stability of test particles in HD 74156. The stable region located at $a = 0.6$ AU, $0.1 < e < 0.25$ is so narrow that an additional companion is very unlikely. Note that the band arises at nonzero eccentricity.

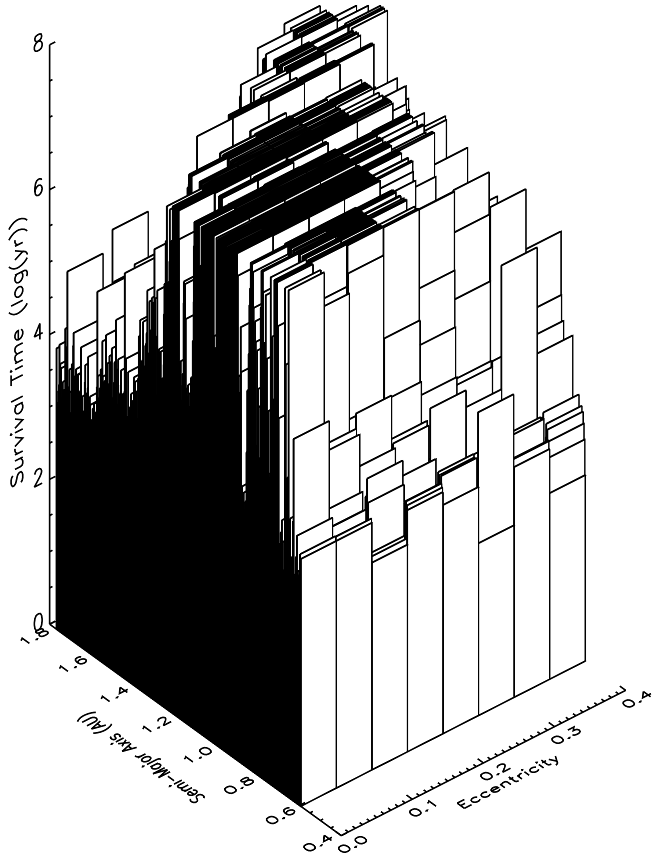

FIG. 3.—Stability of test particles in HD 37124. The most likely orbit for an additional companion lies just interior or exterior to 1 AU, with an eccentricity of ~0.15. This plot also shows the possible orbits of asteroids should no additional planet be present.

complete systems of HD 74156 and HD 168443, and in § 3.2 we analyze the HD 37124, HD 38529, and 55 Cnc systems, which contain zones of stability.

### 3.1. *Complete Systems: HD 74156 and HD 168443*

From Table 1 nothing about these two systems would indicate a priori that they would have no stable zones, although their planetary masses are some of the largest of this subset of systems. Note, however, that HD 38529c is the second-largest planet examined, and it has one of the largest zones of stability (see § 3.2). The ratios of the periods, $R$ (not from the semimajor axes, which appear to not exactly correspond to the periods), of HD 74156 and HD 168443 are 51.4 and 30.5, respectively, which are not the smallest values among separated systems. Nonetheless, these systems show little evidence for stable particles between the currently known planets.

In HD 168443, $R = 30.5$, and we need 786 test particles to fill the region between planets b and c. In Figure 1, we see that regardless of eccentricity or semimajor axis, no test particles survive for even 2 million yr. We therefore conclude that this system is complete out to planet c, that there can be no asteroid belt in this system, and that radial/astrometric surveys should sample this system sparsely.

The best-fit orbital elements for HD 74156 have fluctuated throughout the past several years. The elements presented in Table 1 are from the discovery Web site and date to 2002 May. The elements changed dramatically in 2002 August (namely, $a_c$ increased to 3.8 AU). The data have recently been published (Naef et al. 2004), and the elements changed to values similar to those of 2002 May. Although the system presented here is slightly different than the current best fit, our work is similar enough that any differences should be negligible.

HD 74156 consists of two planets separated by nearly a factor of 40 in period. We need 822 test particles for this system. The eccentricities and masses are lower than in HD 168443. In Figure 2 we present the survivability of test particles in this system. A very narrow strip of stability exists at 0.6 AU, with $e > 0.1$. Therefore any asteroids that may exist in this system must be on significantly eccentric orbits. Although it appears unlikely that any planet could exist in the putative region, we will show in Paper II (S. N. Raymond & R. Barnes 2004, in preparation) that Saturn-mass companions can survive in the system for at least $10^8$ yr.

### 3.2. *Candidate Systems: HD 37124, HD 38529, and 55 Cnc*

Three systems show broad regions of stability for test particles. The current orbital parameters of HD 37124, HD 38529, and 55 Cnc are presented in Table 1. HD 37124 and HD 38529 are classical separated systems, whereas 55 Cnc is a resonant system with a distant, separated companion. In 55 Cnc the outer two planets have $R = 121$, the two planets in HD 37124 have $R = 12.7$, and those in HD 38529 have $R = 152$.

In Figure 3 we show the stable zone for HD 37124. For this system $R = 12.7$ and we integrated 588 test particles. Although dotted with ejections due to high-order resonances, there do appear to be significant regions of stability, most notably at a semimajor axis of approximately 1 AU, near $0.15 \leq e \leq 0.2$. We note that the stable region is not largest at $e = 0$. Therefore, we suggest that the most likely orbit for an additional

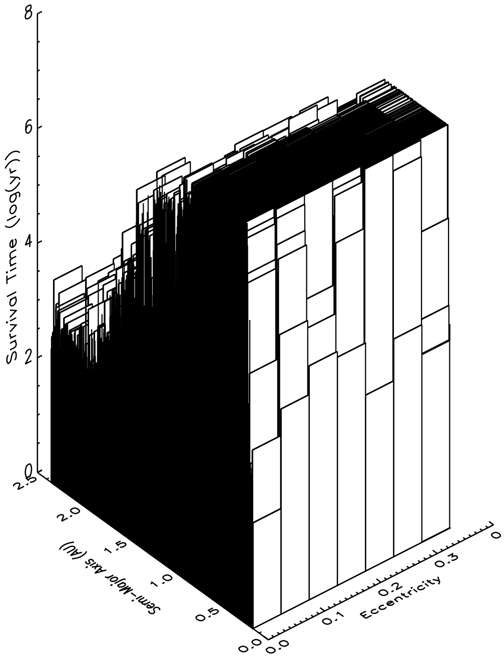

Fig. 4.—Stability of test particles in HD 38529. Stability is most likely in the region $0.25\ \mathrm{AU} \lesssim a \lesssim 0.75$ AU. Above $e = 0.3$ the probability of survival decreases dramatically.

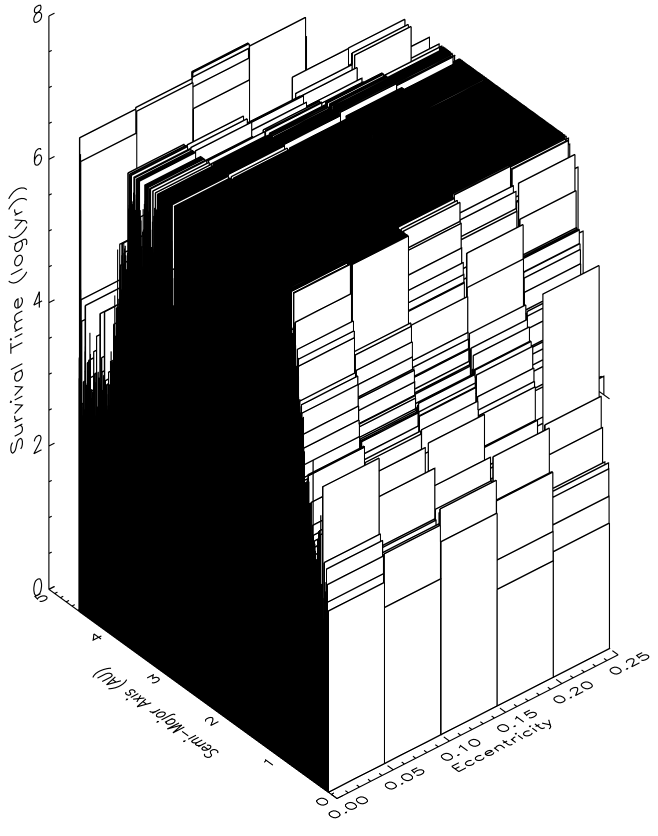

Fig. 5.—Stability of test particles in 55 Cnc. A 1.9 AU wide region of stability exists in this system. Unlike other systems, high eccentricity does not appear to promote instability. This system seems the best candidate for additional companions.

companion is at a semimajor axis slightly smaller or larger than 1 AU (which corresponds to the 5:2 resonance with planet b) and an eccentricity near 0.15. Should there be no planet here, these results suggest that the presence of an asteroid belt in this region of phase space is likely.

Note also in Figure 3 that an additional band of stability arises at $a \approx 1.2$ AU above $e = 0.1$. A third band is visible at $a = 0.9$ AU, $e = 0.15$. A fourth band arises at $a \approx 1.7$ AU at $e > 0.3$. This suggests that the significant (i.e., larger than 0.1) values of extrasolar planetary eccentricities may not be detrimental to system stability. This phenomenon results from the test particle's forced eccentricity (as well as $\varpi$, $i$, and $\Omega$) approaching a planet's eccentricity as $a_{\mathrm{tp}} \rightarrow a_{\mathrm{planet}}$, the test particle's and planet's semimajor axes, respectively (Murray & Dermott 1999). These moderate eccentricities may encourage stability, but at some point in every system, a critical eccentricity is reached and the system becomes unstable. For HD 37124 this threshold depends on $a$. For $a \sim 0.8$ AU the critical eccentricity is 0.25, but for $a \sim 1.6$ AU, there are still stable particles with $e$ as large as 0.4.

As mentioned in § 2, Menou & Tabachnik (2003) obtained a different result for this system. They integrated the region from 0.6 to 1.2 AU for $10^6$ yr, with test particles separated by 0.006 AU. They found that no test particles with eccentricities close to 0.05 survived in this region. The discrepancy results from their stringent criteria for ejection, such as labeling test particles that cross the boundary of the habitable zone as unstable. We find test particles in our habitable zone (in semimajor-axis space) often have significant eccentricities and most likely crossed the boundary at some point during the simulation.

Figure 4 shows the stable regions of HD 38529. $R$ for this system is 152; hence we integrated 1092 test particles. This region also contains a narrow zone of stability. In this system we see that stability lies in between 0.25 and 0.75 AU, and $e < 0.3$. The interior edge of this stable zone is quite sharp, probably because the mass of the inner planet is 15 times smaller than that of the outer. This system has a larger zone of stability than HD 37124, so it seems more likely that a planet might exist in this system.

55 Cnc is the only system with a resonant pair and a distant companion. Of all the systems examined here, $R$ is the largest. Because of this wide separation, 55 Cnc requires 2164 test particles and we could only integrate the system for 5 million yr due to resource constraints. In Figure 5 we plot survivability in this system. Not surprisingly, this system shows the broadest range of stability for test particles. For all eccentricities particles appears to be stable from 0.9 to 2.8 AU. From 2.8 to 3.5 AU the stable regions are broken up by mean-motion resonances with planet d. There does appear to be a slight preference for low eccentricities in this system. Therefore, we suggest that a planet might exist close to 2 AU (the center of the stability zone) with $e \lesssim 0.1$. As before, we also suggest that this region might harbor an asteroid belt or terrestrial planets if it does not contain a gas giant.

55 Cnc was also examined by Rivera & Haghighipour (2003). They examined test particles between planets c and d for 5 million yr, but all the test particles had zero eccentricity. They find a similar stable region. This is also consistent with the results of Marcy et al. (2002), who stated that an additional planet at 1 AU would be stable. Rivera & Haghighipour (2003) also examined the region exterior to planet d and found that particles in this region were stable beyond 10 AU. Given the similarity between our results and theirs in between the extant planets, we would probably obtain a similar result, even for nonzero eccentricities, in the region beyond the outermost planet.

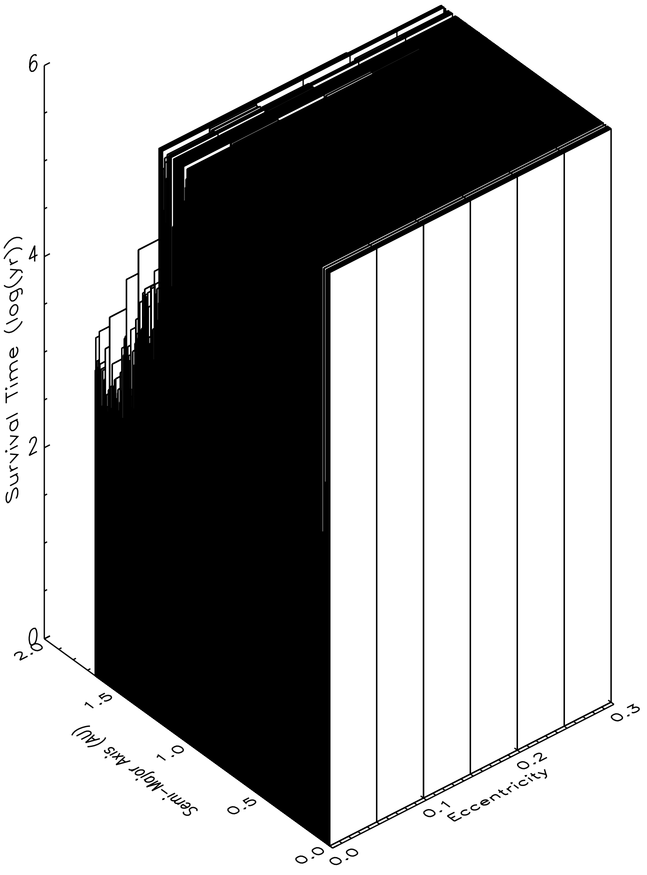

Fig. 6.—Stability of test particles in $\upsilon$ And. This system shows stability between 0.1 and 1.2 AU. Note also that the system is more stable at eccentricities larger than 0.2. The current values of $a_c$ and $e_c$ are 0.8 and 0.24 AU, respectively.

Although there is a broad region of stability, this is only true of massless particles. The resonant pair in this system lies close to instability (B. Henderson & R. Barnes 2004, in preparation). Therefore an additional perturbing mass may increase the eccentricities of the known planets or perturb the system away from perfect resonance and possibly destabilize the system. In Paper II we will address this issue when we place massive companions in this region.

## 4. A CONTROL: $\upsilon$ ANDROMEDAE

To test our methodology, we simulated $\upsilon$ And to determine whether we could predict where the middle planet (planet c) might lie. In Table 1 we present the orbital elements of the planets in this system. In our three-body model, $R = 284$ and we required 814 test particles. The region between planets b and d proved so stable that we were only able to integrate this system for 1 million yr because of CPU time limitations. The stability of this system is shown in Figure 6. As we can see from this figure, we do indeed find that planet c lies in the stable region.

Test particle simulations of $\upsilon$ And were also performed by Rivera & Lissauer (2000). Their simulations ran for 5 million yr. They used all three planets but did place test particles between planets b and c, in addition to particles beyond the orbit of planet d. They found a very narrow region of stability between planets b and c of approximately 0.35 AU in width. They too find a sharp stability edge near planet b. Our hypothesis should therefore suggest that an additional planet may lie in this region between planets b and c as well (see § 5). They also inserted an additional $1M_J$ planet into this system and find that the four-planet system is stable (Rivera & Lissauer 2001). Therefore it is not too surprising that we find such a large stable region in this system. The $\upsilon$ And system is therefore incomplete as observed and is a prime candidate for testing the PPS hypothesis.

TABLE 2

Results of Test Particle Simulations

| System | $R$ | $N$ | $\Delta a$ | $\Delta e$ |
|---|---|---|---|---|
| HD 168443 | 30.5 | 786 | ... | ... |
| HD 74156 | 51.43 | 1004 | ... | ... |
| HD 38529 | 152.0 | 1092 | (0.27, 0.82) | (0.0, 0.3) |
| HD 37124 | 12.7 | 588 | (0.895, 1.11) | (0.0, 0.25) |
| 55 Cnc | 121.0 | 2164 | (0.72, 2.77) | (0.0, 0.25) |
| $\upsilon$ And | 284 | 814 | (0.1, 1.2) | (0.0, 0.3) |

## 5. DISCUSSION AND CONCLUSIONS

We have argued that additional planets should exist in separated systems so that they too lie close to instability. We have tested the PPS hypothesis by integrating the orbits of a large number of massless test particles in five known extrasolar planetary systems. The results of these simulations are summarized in Table 2. In this table $N$ is the number of test particles used and $\Delta a$ and $\Delta e$ are the approximate boundaries wherein an additional companion might exist. We find that some systems cannot contain additional planets (HD 74156 and HD 168443), while others have significant stable zones (HD 37124, HD 38529, and 55 Cnc). From our control experiment, we see that Figure 6 most resembles Figure 5, again suggesting that 55 Cnc is the most likely candidate for an additional companion.

We must note one ambiguity in the PPS hypothesis. As has been shown in other work (Barnes & Quinn 2001; BQ04), $\upsilon$ And is already near the edge of stability. Perhaps an additional companion was in this system but was dynamically unstable and ejected. This left c and d on interacting orbits, and a small annulus of stability between b and c. Planetary systems with any two planets on the edge of stability may negate the possibility of predicting additional companions. They are already on the edge and hence need no additional companions to push them there. This is especially relevant in 55 Cnc. The resonant pair is on the edge of stability (B. Henderson & R. Barnes 2004, in preparation), so perhaps the large gap between planets c and d is irrelevant. However, there is an additional complication in comparing $\upsilon$ And and 55 Cnc. Namely, they may have completely different formation histories. Resonant systems most likely formed from resonant capture during the migration epoch (Snellgrove et al. 2001), whereas $\upsilon$ And may have formed from a large scattering event (Rasio & Ford 1996; Malhotra 2002; BQ04). If a planetary system needs only two planets near the edge of stability to be "packed," then 55 Cnc is already packed and has no additional companions.

Here we have suggested that additional planets exist in between the known planets. It could be, though, that additional planets lie beyond these in an interacting configuration. The work of Rivera & Lissuaer (2000) shows that this is the case for the $\upsilon$ And system (see § 4). If the separated systems presented here actually have more distant companions on interacting orbits, these systems would still lie close to the edge of stability. The $\upsilon$ And system suggests that two interacting planets may be enough to prevent the need for a packed system. Future observations will determine whether planetary systems contain as many planets as they can dynamically balance, or whether they just need at least one interacting or resonant pair.

To further explore the PPS scenario, we will integrate the full four- or five-body systems with a massive planet in the regions of stability defined here. This work will be Paper II. In addition, we will explore the formation of terrestrial planets in these systems in Paper III. In these papers we will show more evidence that the predicted planets can survive for upward of 100 million yr.

Of course stable regions do not guarantee stability, but it is nonetheless intriguing to postulate their existence. Additional planets necessarily change the secular dynamics of a planetary system. Hence test particles alone (which do not alter secular frequencies) cannot adequately predict the orbits of planet-mass particles (e.g., Lissauer et al. 2001). If the candidate systems do contain additional companions, then we strengthen the theory that planet formation is an efficient process and add this as a new requirement to models of planet formation. Whether or not this may break the degeneracy between the so-called core accretion model (i.e., Pollack et al. 1996; Bodenheimer et al. 2000) and the gravitational collapse model (i.e., Boss 2002; Mayer et al. 2002) remains to be seen.

The radial velocity surveys are providing a rich source of knowledge for the field of planet formation, altering our understanding of our own solar system, and changing perspectives of our place in the universe. The correct prediction of a new planet would represent a major step toward understanding the mechanisms of planet formation.

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[5] CONDOR is publicly available at http://www.cs.wisc.edu/condor.

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